OPTIMUM DYNAMIC SYNTHESIS OF SINGLE DEGREE OF FREEDOM PLANAR LINKAGES

BY

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To my loves, Cinha, Ina, Dani

and the unborn one.

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NOTATION

The reading of section 2.1 is recommended for a complete understanding of the notation below.

O = origin of U-V global coordinate system

i = imaginary number ($i^2 = -1$) associated with the V-axis when shaking force is considered

$\underline{i}, \underline{j}, \underline{k}$ = orthogonal unit vectors

$c_{pq} = \cos\phi_{pq}$

$s_{pq} = \sin\phi_{pq}$

ω_i = angular velocity of the input link

m_{pq} = mass of link pq

$\bar{x}_{pq}, \bar{y}_{pq}$ = local coordinates of the center of mass of link pq

$\bar{k}_{pq}$ = radius of gyration of link pq about the center of mass

$x^o_{pq}, y^o_{pq}, k^o_{pq}$ = lumped mass parameters of link pq (section 2.10)

a_{pq} = fixed length of link pq

$x_{rpq}, y_{rpq} = x_r \div a_{pq}$ and $y_r \div a_{pq}$, respectively

$\bar{I}_{pq}$ = mass moment of inertia of link pq about the center of mass

$I_{\underline{p}q}$ = mass moment of inertia of link pq about point "p"

${}^{s}\underline{F}_{pq}$ = shaking force acting <u>on a linkage</u> due to the inertia of link pq

${}^{s}\underline{F}$ = shaking force acting <u>on a linkage</u> due to all the inertias

${}^{s}M_{pq}$ = shaking moment about "O" acting <u>on a linkage</u> due to the inertia of link pq

${}^{s}M$ = shaking moment about "O" acting <u>on a linkage</u> due to all the inertias

${}^{I}T_{pq}$ = input torque of a linkage due to the inertia of link pq

${}^{I}T$ = input torque of a linkage due to all the inertias

$\underline{F}_{pq}$ = reaction force acting on link pq at point "p"

Abstract of Dissertation Presented to the Graduate School of the University of Florida in Partial Fulfillment of the Requirements for the Degree of Doctor of Philosophy

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This dissertation describes an approach to the optimization of the dynamic behavior of single degree of freedom planar linkages, with known geometry and motion, via the digital computer. The first phase of this approach consists of the dynamic synthesis of one or two of the dynamic properties considered in this work, i.e., shaking force, shaking moment, and input torque. The result of synthesis is a fruitful reduction in the number of free design variables. The second phase is essentially an optimization procedure where the values of the free parameters remaining from synthesis are determined such that the bearing reactions and the dynamic properties not considered in the synthesis phase are optimized.

An interactive CAD package for the optimization of the dynamics of four-bar linkages is presented, and two examples based on the same mechanism are considered. The computer programs written in APL language are contained in two appendices.

CHAPTER I
INTRODUCTION

1.1 Purpose

Dynamics of mechanisms and machinery has become a field crucially important due to the increasing demand for machines with higher speeds and higher precision of operation. Thus, there is a growing need for efficient computer aided design (CAD) techniques to optimize the dynamic behavior of such machines. As an attempt to contribute to the fulfillment of this need, the objective of this work is to meliorate the dynamic synthesis method presented by Elliott in Ref. [1], and then use it as a basis for an interactive computer package for the optimization of the shaking force, shaking moment, input torque, and reaction forces of planar four-bar linkages.

1.2 History and Literature Review

Mechanisms are used everywhere in the modern world; from simple everyday mechanical items to complex machines:

-- toys, photographic cameras and cassette recorders

-- household items

-- agriculture, printing and textile machinery

-- packaging and assembly machinery

-- machine tools and presses

-- vehicles and spacecrafts

-- prostheses, robots and industrial manipulators, etc. . . .

Although this list is far from being complete, it is sufficient to illustrate the great technological importance that the engineering field of mechanism and machine design has in modern life.

The high level of today's mechanisms and machines is a result of man's old desire to understand and control the laws of nature and his intrinsic spirit of optimization which has made him search for better ways of doing things since his first appearance on earth.

The evolution of mechanisms and machines practically parallels the historic development of mankind [2,3,4]. The earliest records of the invention and use of simple mechanical devices are the prehistoric drawings, tools, weapons, traps, and other implements discovered by archaeologists. These simple devices were created by the trial and error method of experimentation. In fact, this method remained for many centuries as the only approach adopted by the ancient inventors until science sporadically started to change this situation.

When and where science actually began is difficult to determine [2,5]. The ancient Egyptians probably were, to

a certain extent, familiar with the basic laws of mechanical equilibrium without which the pyramids and other grand structures could not have been built. The "science which is concerned with the motions and equilibrium of masses" was christened "mechanics"[1] by the great Greek philosopher Aristotle (384-322 B.C.) who summed up in his Physics the facts of mechanics known to the ancients; however, his basic law of motion of bodies was incorrect [2,3], which was found only 19 centuries later by the Italian Galileo Galilei (1564-1642) who was the first to formulate the law of falling bodies. Proceeding from Galileo's work and from the achievements of his contemporaries, the great English scientist Sir Isaac Newton (1642-1727) discovered the fundamental laws of mechanical motion and the universal law of gravitation and stated them in a very clear and concise form in his Principia (1687) [6].

Galileo, Newton and many other scientists of the sixteenth and seventeenth centuries laid down the scientific foundation on which a new engineering was built. Until then, engineering had been an art based on empirical rules passed from one generation to another and derived very much from trial and error experiments [4,7].

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1 The quoted definition of the science of mechanics was given by the Austrian physicist Ernst Mach (1838-1916) [2, p. 3].

During the eighteenth and nineteenth centuries, the era marked by the industrial revolution and the triumph of the machine, science and engineering both grew rapidly [4,8]. Several new branches of the science of mechanics were created. In 1875, the German Franz Reuleaux (1829-1905) set the basis for mechanism science with his pioneer book Theoretisch Kinematik which was translated into English in 1876 [9] and, in 1888, L. Burmester (1840-1927) published his classical book Lehrbuch der Kinematik [10]. The Russian school of mechanism science was also created in this period by P.L. Chébyshev (1821-1894).

These two schools, the German and the Russian, were greatly developed during the first half of the twentieth century. Although this short period was marked mainly by the growth of these two schools, there were also many significant contributions to this field from other parts of the world, e.g., Allievi (Italy), Rosenauer and A. H. Willis (Australia), Poeschl and Federhofer (Austria), Nicaise, Koenings, Bricard, and Manheim (France).

In the United States, it was not until 1940 that Svoboda [11] developed numerical methods to design a four-bar linkage to generate a desired function. In 1951, the publication by Hrones and Nelson [12] of an "Atlas" containing approximately 10,000 coupler curves offered a very practical approach for some design engineers. Undoubtedly, the publications from Prof. F. Freudenstein

in the fifties played a crucial role in the development of mechanism science in the U.S. The research activity related to mechanisms has accelerated a great deal during the last three decades, particularly in the United States.

To date, the scientific and engineering field of mechanisms has accumulated a voluminous amount of knowledge. As implied in the above historical overview, this is due to the work of creative minds from many different parts of the world. A great deal of this knowledge is concentrated in the subfield of "Kinematics" the importance of which cannot be overemphasized; a mechanism with poor kinematic characteristics will not perform satisfactory. However, as stated in the beginning of this chapter, high speed and high precision of operation cannot be achieved by a machine with poor dynamic behavior.

The number of investigations into the balancing of mechanisms has increased considerably during the last two decades. One of the earliest works in this period was done in 1966 by Sherwood [13] who suggested a method for minimizing the fluctuation of kinetic energy of a four-bar linkage by using the principle of dynamically equivalent systems. Chi-Yeh Han [14] presented, in 1967, a method for balancing the shaking force and shaking moment of a punch-and-reader unit by means of a single counterweight attached to the input crank with certain phase angle.

In 1968, Lowen and Berkof [15] performed a survey of the literature about force and moment balancing for mechanisms with rigid body links. In that same year, Kamenskii [16] classified the various known balancing methods and established quality criteria for static, dynamic, and combined balancing of certain mechanisms. Still in 1968, Gheronimus [17] applied Chébyshev's methods of best approximation of functions to the balancing of mechanisms.

In 1969, Berkof and Lowen [18] introduced the method of "linearly independent vectors" by forcing the center of mass of four- and six-bar linkages to stay stationary. This method became the inspiration for several following works. Also in 1969, Sherwood and Hockey [19] extended the optimization method suggested by Sherwood in 1966 [13], by making it more general.

Skreiner [20], in 1970, discussed how the process of mechanism design brings together in one work the various considerations usually treated one at a time in the literature.

In 1971, Benedict and Tesar [21] showed how to calculate the dynamic response of mechanical systems by means of "kinematic influence coefficients"; and Berkof and Lowen [22] applied the least-square theory to the optimization of the shaking moment of a fully force-balanced four-bar linkage.

In 1972, Hockey [23] proposed a technique, based on previous works [13,19,24], for the minimization of the fluctuation of input torque in planar mechanisms under the influence of external loads. Tepper and Lowen [25] stated certain general theorems concerning full force balancing of planar mechanisms by redistribution of internal mass.

In 1973, Tepper and Lowen [26] introduced the "theory of isomomental ellipses" and showed that the RMS shaking moment of unbalanced planar mechanisms is constant with respect to all points along certain concentric ellipses. Berkof [27] demonstrated how the use of counterweights and a physical pendulum link can provide complete force and moment balance of an inline four-bar linkage. Stevensen [28] pointed out that "very little of the developments for the balancing of mechanisms may be used directly in the balancing of machines," and presented a general method for the complete balancing of a harmonic unbalance of any machine.

In 1975, Paul [29] presented a computer oriented overview of the various techniques available for generation of the equations of motion of linkages, for integrating them numerically, and for calculating the bearing reactions.

Matthew and Tesar [30,31], in 1977, considered the synthesis of spring parameters to satisfy specified energy

levels and to balance general forcing functions in planar mechanisms.

Benedict and Tesar [32], in 1978, made a significant contribution to mechanism science. They presented a formulation based on velocity and acceleration influence coefficients which has proven to be very efficient for kinematic and dynamic analysis of multi-degree of freedom mechanisms.

Walker and Oldham, in 1978 [33] and 1979 [34], considered the balancing of frame forces in planar linkages possibly containing prismatic joints and presented a method for checking whether a linkage can be fully force-balanced using counterweights alone.

In 1981, Elliott and Tesar [35] introduced a general theory, extracted from [1], that allows complex planar mechanisms to attain pre-specified values of any conceivable dynamic property. As declared in section 1.1, that theory will be enhanced in this work in order to make it more easily applicable to the synthesis of shaking force, shaking moment, and input torque of such mechanisms. Still in 1981, Tricamo and Lowen [36,37] presented a concept for reducing the residual shaking forces of theoretically fully force balanced mechanisms, and developed an experimental balancing machine that reduced the actual shaking force components of a four-bar linkage by more than fifty percent.

In 1982, Tricamo and Lowen [38] showed how the maximum shaking force of a four-bar linkage can be lowered to a pre-specified value by shrinking the force hodograph of the mechanism with the help of two counterweights. In that same year, they [39] extended the previous concept by using a third counterweight to simultaneously minimize the maximum values of input torque, shaking moment and bearing reactions while obtaining the prescribed maximum value of the shaking force.

Freeman and Tesar [40] presented in 1982 a technique that allows the selection of any generalized coordinate(s) of a complex mechanical system to be the reference parameter(s) for the development of the system's dynamic model. Once this model is obtained, it can be transferred to any other reference parameter(s) by simple transformation equations.

The works [41-57] represent some of the other contributions to the field of dynamics of planar mechanisms during the past decade.

In the nineteenth century, the industrial revolution considerably enhanced man's physical powers. In the present century, a second industrial revolution is taking place, with computers offering an enhancement of man's mental capabilities. The use of computers in engineering, particularly in the field of mechanisms, has been rising exponentially. During the last decade, several

general-purpose computer programs for analysis and design of mechanisms have been developed and described in the literature. Notable among these are IMP [58], DAMN [59], DRAM [60], ADAMS [61], DADS [62], DYMAC [63], KINSYN [64], MECSYN [65], LINCAGES [66], RECSYN [67], ANIMEC [68], MEDUSA [69], ISD-FORSS-II [70], VECNET [71], PLANET [72], and SPACEBAR [73].

CHAPTER II
DYNAMIC SYNTHESIS

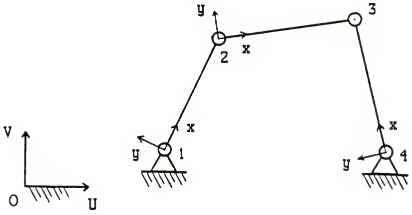
The objective of this chapter is to provide a solid foundation for the effective dynamic synthesis of complex planar linkages, and then derive the formulations pertinent to the dynamic synthesis of the four-bar, the slider-crank, the Watt I and II, and the Stephenson I, II and III mechanisms.

2.1 Preliminary Remarks

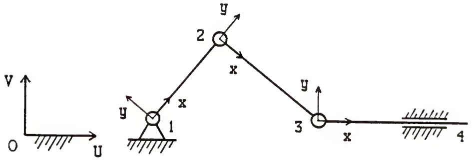
This work is concerned only with mechanisms having rigid links. Hence, the distance between any two given points of a link will always be considered invariant in the derivations to follow.

An x-y Cartesian coordinate system will be conveniently fixed to every moving link of a mechanism, as shown by the examples given in Fig. 2.1. Accordingly, the following comments are appropriate at this point:

1. An ordered pair of natural numbers will be used to designate a moving link and additionally indicate the location of the x-y coordinate system of that particular link. For example, the origin of the x-y coordinate system of representative link 62 of



(a)



(b)

Figure 2.1 Examples of Coordinate System Placement

a certain mechanism is attached to point 6 of that link, and the positive x-axis passes through point 2 of the same link.

2. A general moving link will be designated by "pq," where "p" and "q" have the above meaning.
3. The x-y coordinate system of link pq will generally be referred to as its "local" coordinate system.
4. The U-V coordinate system fixed to the ground of the mechanism will analogously be called the "global" coordinate system.

2.2 Procedure to Locate the x-y Coordinate Systems

When choosing the locations for the x-y coordinate systems of the moving links of a mechanism, some care should be taken so that the method contained in this work can be adequately applied. The justifications for this will be given either explicitly or implicitly in this chapter, as they become opportune. For now, it is desirable to have some guidelines indicating how to properly perform the above task.

Any moving link (binary, ternary, quaternary, etc.) and its local coordinate system should correspond to one of the three cases shown in Fig. 2.2. As a complement to that figure consider the following remarks:

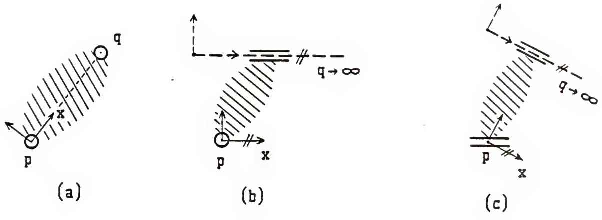


Figure 2.2 The Three Types of Local Coordinate System Placement

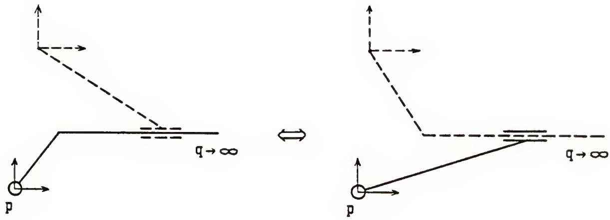


Figure 2.3 Equivalence of Representations for a Link pq With Prismatic Joint

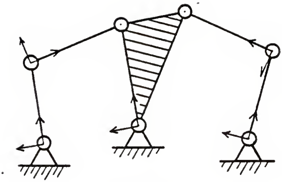
1. The origin of the local coordinate system should be attached to the center of a revolute joint, if there is one.
2. The zero angle between the x-axes of links pq and rq in cases (b) and (c) yields simpler analytical expressions.
3. Since it is unimportant to this work to know which link is sliding internally in a prismatic pair, the two representations in Fig. 2.3 are equivalent.

Given a linkage mechanism, the placement of the x-y coordinate systems must be made considering the whole linkage, and the following rules-of-thumb should be satisfied:

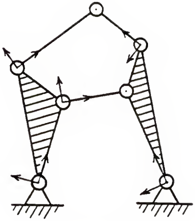
Rule 1: If a link has a grounded pin joint, the origin of its local coordinate system should be attached to the center of that joint. However, if the grounded joint is prismatic, the x-axis should be parallel (or coincident) to the axis of that sliding joint (Fig. 2.1b).

Rule 2: Each revolute joint of a linkage should contribute to the placement of the x-y coordinate system of at least one of the links jointed by it.

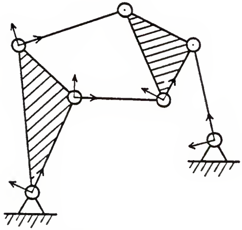
It should be noticed that for all the mechanisms shown in Fig. 2.4, Rule 2 is automatically fulfilled if Rule 1 is satisfied. Analogously, only Rule 1 has to be considered for mechanisms composed only of binary links (Fig. 2.1). On the other hand, the consideration of



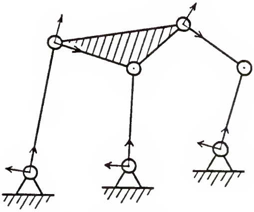
Watt II



Stephenson I



Stephenson II



Stephenson III

Figure 2.4 Six-Bar Mechanisms Satisfying Rules 1 and 2

Rule 2 becomes important for some mechanisms, as, for example, the six- and eight-bar linkages illustrated in Fig. 2.5.

2.3 Mass Parameters and Motion Specification of Moving Links

Figure 2.6 shows the general representation of a link with planar motion which will be used throughout this dissertation. Four "mass parameters" will be associated with every moving link, i.e., the mass m_{pq}, the radius of gyration $\bar{k}_{pq}$ about the center of mass, and the coordinates $(\bar{x}_{pq}, \bar{y}_{pq})$ of the center of mass.

To apply the formulations of this chapter in the dynamic synthesis (or analysis) of a linkage, it is necessary first to know its motion up to the second derivative at every considered instant of time. This condition can be satisfied if, for every moving link, the position, velocity, and acceleration of points "p" and "q" are known or, alternatively, if ϕ_{pq}, $\dot{\phi}_{pq}$ and $\ddot{\phi}_{pq}$ are known in addition to the motion of "p" (Fig. 2.6).

The shaking force, shaking moment, and input torque of a linkage due to its inertia can be calculated by adding the individual contributions from each moving link. The next sections will deal with these dynamic properties for the two cases indicated in the above paragraph. First, expressions for links with "p-ϕ motion

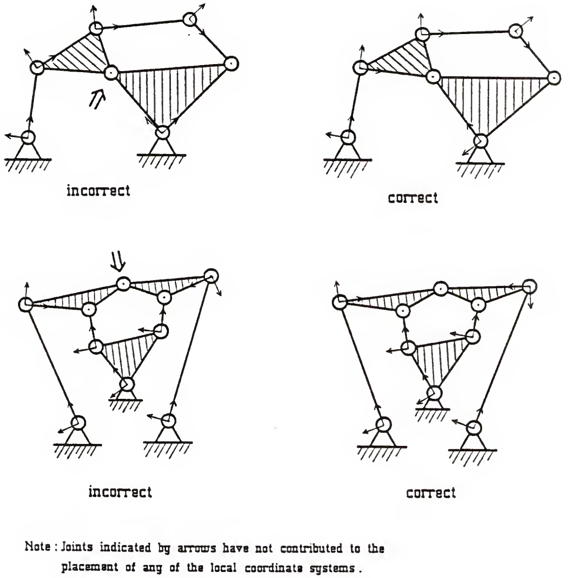


Figure 2.5 Watt I and an Eight-Bar Mechanism Illustrating Rule 2

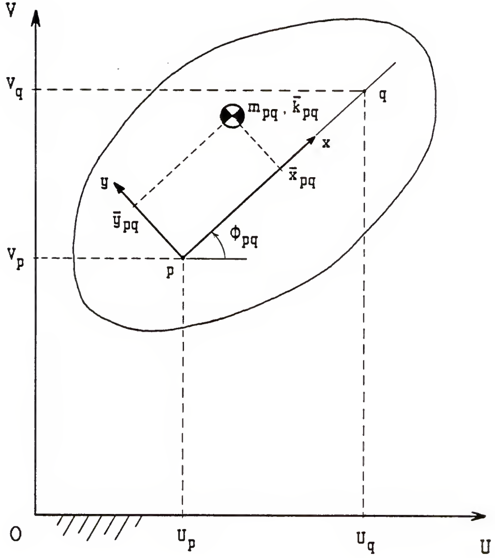


Figure 2.6 A Generic Link With Planar Motion

specification" will be derived, then links with "p-q motion specification" will be considered.

These expressions will be written in a form suitable for dynamic synthesis. They are essentially the same expressions derived algebraically by Elliott and Tesar [1,35]. The objective here is to develop a better insight into their formulations to apply them effectively in the rest of this work.

2.4 Inertial Shaking Force (Links With p-ϕ Motion Specification)

The shaking force acting on a linkage due to the inertia of link pq can be expressed as (Figs. 2.6 and 2.7).

$$
\begin{aligned}
{}^{s}\underline{F}_{pq} &= m_{pq}(\ddot{\underline{r}}_{p}+\ddot{\underline{r}}_{G/p}) = m_{pq}(\ddot{U}_{p}+i\ddot{V}_{p}) + \\
&+ m_{pq}\bar{x}_{pq}\ddot{\phi}_{pq}(-s_{pq}+ic_{pq})-m_{pq}\bar{y}_{pq}\ddot{\phi}_{pq}(c_{pq}+is_{pq}) - \\
&- m_{pq}\bar{x}_{pq}\dot{\phi}_{pq}^{2}(c_{pq}+is_{pq})+m_{pq}\bar{y}_{pq}\dot{\phi}_{pq}^{2}(s_{pq}-ic_{pq})
\end{aligned}
$$

where $c_{pq} = \cos\phi_{pq}$ and $s_{pq} = \sin\phi_{pq}$.

Re-grouping the terms results in

$$
{}^{s}\underline{F}_{pq} = Y^{1}_{1pq}D^{1}_{1pq}+Y^{1}_{2pq}D^{1}_{2pq}+Y^{1}_{3pq}D^{1}_{3pq} \qquad (2.4.1)
$$

where,

$$
Y^{1}_{1pq} = m_{pq}
$$

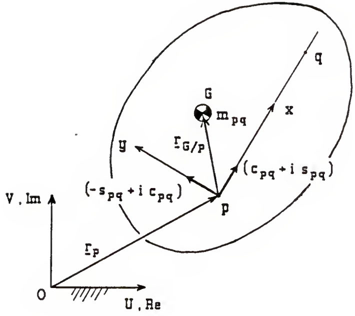


Figure 2.7 Diagram of a Moving Link for Shaking Force Derivation

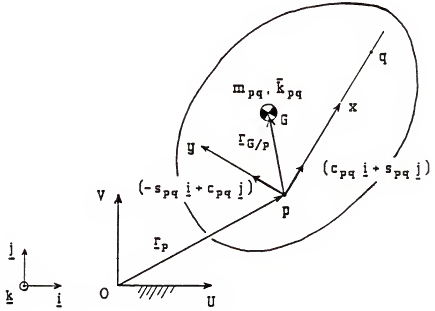


Figure 2.8 Diagram of a Moving Link for Shaking Moment and Input Torque Derivations

$$D^1_{1pq} = \ddot{U}_p + i\ddot{V}_p$$

$$Y^1_{2pq} = m_{pq}\bar{x}_{pq}$$

$$D^1_{2pq} = \ddot{\phi}_{pq}(-s_{pq}+ic_{pq}) - \dot{\phi}^2_{pq}(c_{pq}+is_{pq})$$

$$Y^1_{3pq} = m_{pq}\bar{y}_{pq}$$

$$D^1_{3pq} = -\ddot{\phi}_{pq}(c_{pq}+is_{pq}) + \dot{\phi}^2_{pq}(s_{pq}-ic_{pq})$$

Equation (2.4.1) was considered free of any linear dependency in Refs. [1,35]. However, that is not so since $D^1_{3pq} = iD^1_{2pq}$, and Eq. (2.4.2) must be rewritten as

$$^s\underline{F}_{pq} = Y^1_{1pq}D^1_{1pq} + (Y^1_{2pq} + iY^1_{3pq})D^1_{2pq} \qquad (2.4.2)$$

because no linear dependency between the D-coefficients can be allowed during synthesis.

2.5 Inertial Shaking Moment (Links With p-ϕ Motion Specification)

The Shaking Moment about the origin O acting on a linkage due to the inertia of link pq can be expressed as (Figs. 2.6 and 2.8)

$$^sM_{pq} = (\underline{r}_p + \underline{r}_{G/p}) \times m_{pq}(\ddot{\underline{r}}_p + \ddot{\underline{r}}_{G/p}) \cdot \underline{k} + \bar{I}_{pq}\ddot{\phi}_{pq}$$

$$= m_{pq}(\underline{r}_p \times \ddot{\underline{r}}_p \cdot \underline{k}) + m_{pq}(\underline{r}_{G/p} \times \ddot{\underline{r}}_p \cdot \underline{k}) + m_{pq}(\underline{r}_p \times \ddot{\underline{r}}_{G/p} \cdot \underline{k}) + I_{\underline{p}q}\ddot{\phi}_{pq}$$

$$= m_{pq}(U_p\ddot{V}_p - V_p\ddot{U}_p) +$$

$$+ m_{pq}\bar{x}_{pq}(\ddot{V}_p c_{pq} - \ddot{U}_p s_{pq}) - m_{pq}\bar{y}_{pq}(\ddot{V}_p s_{pq} + \ddot{U}_p c_{pq}) +$$

$$+ m_{pq}\bar{x}_{pq}\ddot{\phi}_{pq}(U_p c_{pq} + V_p s_{pq}) + m_{pq}\bar{y}_{pq}\ddot{\phi}_{pq}(-U_p s_{pq} + V_p c_{pq}) +$$

$$+ m_{pq}\bar{x}_{pq}\dot{\phi}^2_{pq}(-U_p s_{pq} + V_p c_{pq}) - m_{pq}\bar{y}_{pq}\dot{\phi}^2_{pq}(U_p c_{pq} + V_p s_{pq}) +$$

$$+ m_{pq}(\bar{x}^2_{pq} + \bar{y}^2_{pq} + \bar{k}^2_{pq})\ddot{\phi}_{pq}$$

Re-grouping the terms results in

$$^{s}M_{pq} = Y^1_{1pq}D^3_{1pq} + Y^1_{2pq}D^3_{2pq} + Y^1_{3pq}D^3_{3pq} + Y^1_{4pq}D^3_{4pq} \tag{2.5.1}$$

where

$$D^3_{1pq} = U_p\ddot{V}_p - V_p\ddot{U}_p$$

$$D^3_{2pq} = (\ddot{V}_p c_{pq} - \ddot{U}_p s_{pq}) + \ddot{\phi}_{pq}(U_p c_{pq} + V_p s_{pq}) + \dot{\phi}^2_{pq}(-U_p s_{pq} + V_p c_{pq})$$

$$D^3_{3pq} = -(\ddot{V}_p s_{pq} + \ddot{U}_p c_{pq}) + \ddot{\phi}_{pq}(-U_p s_{pq} + V_p c_{pq}) - \dot{\phi}^2_{pq}(U_p c_{pq} + V_p s_{pq})$$

$$D^3_{4pq} = \ddot{\phi}_{pq}$$

$$Y^1_{4pq} = m_{pq}(\bar{x}^2_{pq} + \bar{y}^2_{pq} + \bar{k}^2_{pq})$$

Note: Y^1_{jpq}, j=1,2,3 were defined in section 2.4.

2.6 Inertial Input Torque (Links With p-ϕ Motion Specification)

The torque acting on the input shaft of a linkage due to the inertia of link pq can be expressed as (Fig. 2.6 and 2.8)

$$ {}^{I}T_{pq} = [(\dot{\underline{r}}_p+\dot{\underline{r}}_{G/p})\cdot m_{pq}(\ddot{\underline{r}}_p+\ddot{\underline{r}}_{G/p})+\dot{\phi}_{pq}\bar{I}_{pq}\ddot{\phi}_{pq}]\div\omega_i $$

where ω_i = angular velocity of the input link. Thus,

$$ {}^{I}T_{pq} = [(\dot{\underline{r}}_p\cdot m_{pq}\ddot{\underline{r}}_p)+(\dot{\underline{r}}_{G/p}\cdot m_{pq}\ddot{\underline{r}}_p)+(\dot{\underline{r}}_p\cdot m_{pq}\ddot{\underline{r}}_{G/p})+\dot{\phi}_{pq}I_{\underline{p}q}\ddot{\phi}_{pq}]\div\omega_i $$

$$ = [m_{pq}(\dot{U}_p\ddot{U}_p+\dot{V}_p\ddot{V}_p) + $$

$$ + m_{pq}\bar{x}_{pq}\dot{\phi}_{pq}(-\ddot{U}_p s_{pq}+\ddot{V}_p c_{pq})-m_{pq}\bar{y}_{pq}\dot{\phi}_{pq}(\ddot{U}_p c_{pq}+\ddot{V}_p s_{pq}) + $$

$$ + m_{pq}\bar{x}_{pq}\ddot{\phi}_{pq}(-\dot{U}_p s_{pq}+\dot{V}_p c_{pq})-m_{pq}\bar{y}_{pq}\ddot{\phi}_{pq}(\dot{U}_p c_{pq}+\dot{V}_p s_{pq}) - $$

$$ - m_{pq}\bar{x}_{pq}\dot{\phi}^2_{pq}(\dot{U}_p c_{pq}+\dot{V}_p s_{pq})+m_{pq}\bar{y}_{pq}\dot{\phi}^2_{pq}(\dot{U}_p s_{pq}-\dot{V}_p c_{pq}) + $$

$$ + m_{pq}(\bar{x}^2_{pq}+\bar{y}^2_{pq}+\bar{k}^2_{pq})\dot{\phi}_{pq}\ddot{\phi}_{pq}]\div\omega_i $$

Re-grouping the terms results in

$$ {}^{I}T_{pq} = {}^{1}Y_{1pq}{}^{5}D_{1pq}+{}^{1}Y_{2pq}{}^{5}D_{2pq}+{}^{1}Y_{3pq}{}^{5}D_{3pq}+{}^{1}Y_{4pq}{}^{5}D_{4pq} \qquad (2.6.1) $$

where

$$ {}^{5}D_{1pq} = (\dot{U}_p\ddot{U}_p+\dot{V}_p\ddot{V}_p)\div\omega_i $$

$$ {}^{5}D_{2pq} = [\dot{\phi}_{pq}(-\ddot{U}_p s_{pq}+\ddot{V}_p c_{pq})+\ddot{\phi}_{pq}(-\dot{U}_p s_{pq}+\dot{V}_p c_{pq}) - $$

$$ - \dot{\phi}^2_{pq}(\dot{U}_p c_{pq}+\dot{V}_p s_{pq})]\div\omega_i $$

$$ {}^{5}D_{3pq} = [-\dot{\phi}_{pq}(\ddot{U}_p c_{pq}+\ddot{V}_p s_{pq})-\ddot{\phi}_{pq}(\dot{U}_p c_{pq}+\dot{V}_p s_{pq}) + $$

$$ + \dot{\phi}^2_{pq}(\dot{U}_p s_{pq}-\dot{V}_p c_{pq})]\div\omega_i $$

$$D^{5}_{4pq} = \dot{\phi}_{pq}\ddot{\phi}_{pq} \div \omega_i$$

2.7 Inertial Shaking Force (Links With p-q Motion Specification)

As was shown in [1,35], the shaking force acting on a linkage due to the inertia of link pq can be expressed as

$$^{S}\underline{F} = \Sigma\, Y^{2}_{ipq} D^{2}_{ipq} \; ; \; i = 1 \text{ to } 4 \qquad (2.7.1)$$

where

$$Y^{2}_{1pq} = m_{pq}(1 - \bar{x}_{pq} \div a_{pq}) \qquad (2.7.2.a)$$

$$Y^{2}_{2pq} = m_{pq}(-\bar{y}_{pq} \div a_{pq}) \qquad (2.7.2.b)$$

$$Y^{2}_{3pq} = m_{pq}(\bar{y}_{pq} \div a_{pq}) \qquad (2.7.2.c)$$

$$Y^{2}_{4pq} = m_{pq}(\bar{x}_{pq} \div a_{pq}) \qquad (2.7.2.d)$$

(a_{pq} = length of link pq = distance between points "p" and "q")

and

$$D^{2}_{1pq} = \ddot{U}_p + i\ddot{V}_p \qquad (2.7.3.a)$$

$$D^{2}_{2pq} = -\ddot{V}_p + i\ddot{U}_p \qquad (2.7.3.b)$$

$$D^{2}_{3pq} = -\ddot{V}_q + i\ddot{U}_q \qquad (2.7.3.c)$$

$$D^{2}_{4pq} = \ddot{U}_q + i\ddot{V}_q \qquad (2.7.3.d)$$

Equation (2.7.1) was also regarded in [1,35] as containing no linear dependency. But, since $D^2_{2pq} = iD^2_{1pq}$, and $D^2_{3pq} = iD^2_{4pq}$, it must be modified to

$$ {}^s\underline{F}_{pq} = (Y^2_{1pq}+iY^2_{2pq})D^2_{1pq}+(Y^2_{4pq}+iY^2_{3pq})D^2_{4pq} \qquad (2.7.4) $$

which is now in a form suitable for synthesis.

Clearly, all the Y's above have units of mass, and D^2_{1pq} and D^2_{4pq} are the equivalent accelerations of points "p" and "q," respectively. Therefore, according to Eq. (2.7.4), the mass m_{pq} of link pq can be transferred from the center of gravity to points "p" and "q" as "composite" point masses expressed in complex form. As depicted in Fig. 2.9, this new system of two composite point masses is exactly equivalent to the original link model, for shaking force considerations.

The concept of composite point masses allows one to write the expression of the total shaking force acting on a linkage, in a single step, without linear dependencies, by just looking at the diagram of the mechanism (Fig. 2.10).

The mass m_{pq} at the center of gravity of a link pq was previously transferred to points "p" and "q" by means of the four dimensionless "transfer coefficients" inside the parentheses in Eqs. (2.7.2). Analogously, any point mass, be it composite or not, can be transferred by means

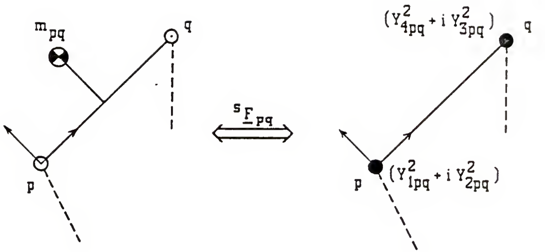


Figure 2.9 Two Shaking Force Equivalent Link Models

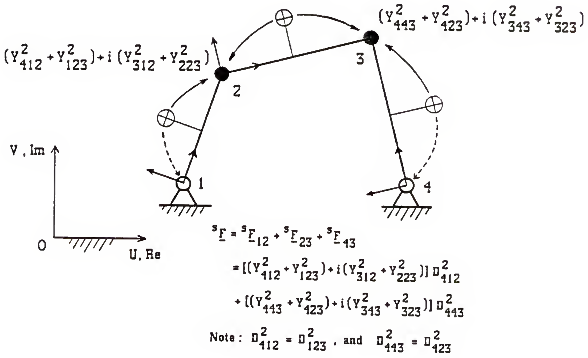


Figure 2.10 Shaking Force Derivation for a Four-Bar Linkage

of the same coefficients if $\bar{x}_{pq}$ and $\bar{y}_{pq}$ are replaced with the coordinates of the point where the mass to be transferred is located. This is exemplified for a composite point mass (A+iB) in Fig. 2.11.

Now it can be understood why rule 2 given in section 2.2 must be satisfied for the six- and eight-bar mechanisms in Fig. 2.5. If that rule is overlooked, the minimum number of composite point masses with linearly independent D-coefficients may not be obtained (Fig. 2.12). Although Fig. 2.12 is based on shaking force only, the same justification holds for shaking moment and input torque, as will become apparent in the following sections.

2.8 Inertial Shaking Moment and Inertial Input Torque (Links With p-q Motion Specification)

As was shown in [1,35], the shaking moment about the origin O acting on a linkage due to the inertia of link pq can be expressed as

$$^{S}M_{pq} = \Sigma\ Y^{3}_{ipq}D^{4}_{ipq}\ ;\ i = 1\ \text{to}\ 4 \qquad (2.8.1)$$

where

$$Y^{3}_{1pq} = m_{pq}[(a_{pq}-\bar{x}_{pq})^{2}+\bar{y}^{2}_{pq}+\bar{k}^{2}_{pq}]\div a^{2}_{pq} \qquad (2.8.2.a)$$

$$Y^{3}_{2pq} = m_{pq}(a_{pq}\bar{x}_{pq}-\bar{x}^{2}_{pq}-\bar{y}^{2}_{pq}-\bar{k}^{2}_{pq})\div a^{2}_{pq} \qquad (2.8.2.b)$$

$$Y^{3}_{3pq} = m_{pq}\bar{y}_{pq}\div a_{pq} \qquad (2.8.2.c)$$

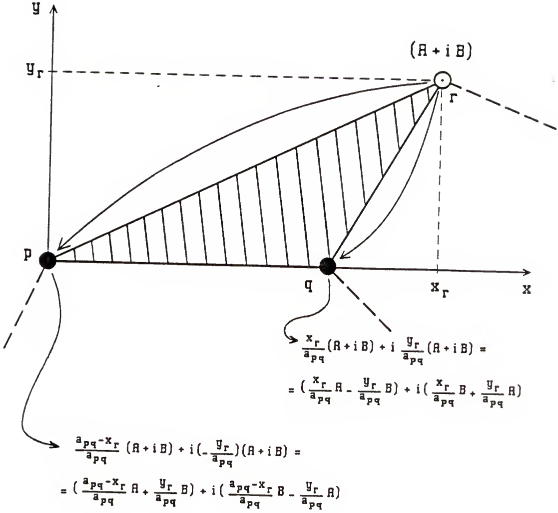


Figure 2.11 Transformation of a Composite Point Mass at "r" into Two Composite Point Masses at "p" and "q"

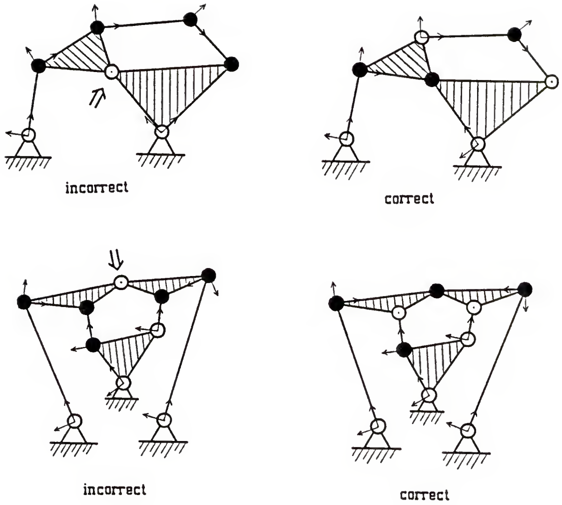


Figure 2.12 Examples of the Influence of Local Coordinate Systems on the Number of Composite Point Masses

$$Y^3_{4pq} = m_{pq}(\bar{x}^2_{pq}+\bar{y}^2_{pq}+\bar{k}^2_{pq}) \div a^2_{pq} \tag{2.8.2.d}$$

and

$$D^4_{1pq} = U_p\ddot{V}_p - V_p\ddot{U}_p \tag{2.8.3.a}$$

$$D^4_{2pq} = (U_p\ddot{V}_q - V_p\ddot{U}_q)+(U_q\ddot{V}_p - V_q\ddot{U}_p) \tag{2.8.3.b}$$

$$D^4_{3pq} = (U_p\ddot{U}_q + V_p\ddot{V}_q)-(U_q\ddot{U}_p + V_q\ddot{V}_p) \tag{2.8.3.c}$$

$$D^4_{4pq} = U_q\ddot{V}_q - V_q\ddot{U}_q \tag{2.8.3.d}$$

It was also shown in [1,35] that the torque acting on the input shaft of a linkage due to the inertia of link pq can be expressed as (Appendix A demonstrates by means of kinematic influence coefficients):

$$^{I}T_{pq} = \Sigma\, Y^3_{ipq} D^6_{ipq} \quad ; \; i = 1 \text{ to } 4 \tag{2.8.4}$$

where

$$D^6_{1pq} = (\dot{U}_p\ddot{U}_p+\dot{V}_p\ddot{V}_p) \div \omega_i \tag{2.8.5.a}$$

$$D^6_{2pq} = [(\dot{U}_p\ddot{U}_q+\dot{V}_p\ddot{V}_q)+(\dot{U}_q\ddot{U}_p+\dot{V}_q\ddot{V}_p)] \div \omega_i \tag{2.8.5.b}$$

$$D^6_{3pq} = [-(\dot{U}_p\ddot{V}_q-\dot{V}_p\ddot{U}_q)+(\dot{U}_q\ddot{V}_p-\dot{V}_q\ddot{U}_p)] \div \omega_i \tag{2.8.5.c}$$

$$D^6_{4pq} = (\dot{U}_q\ddot{U}_q+\dot{V}_q\ddot{V}_q) \div \omega_i \tag{2.8.5.d}$$

A glance at Eqs. (2.8.2) shows that all the Y's have units of mass. Thus, examining Eq. (2.8.1) and Eqs.

(2.8.3-5), it can be concluded that for shaking moment and input torque considerations, the mass at the center of gravity of a link pq and the moment of inertia about that point can be replaced with four point masses located at "p," "q," and two other points whose actual positions are immaterial to this work. It should be noticed though, that point masses Y^3_{2pq} and Y^3_{3pq} are, in general, continuously changing their positions on link pq, as opposed to point masses Y^3_{1pq} and Y^3_{4pq} which are always located at "p" and "q," respectively. This fact is represented by the two broken lines in Fig. 2.13.

It is important to observe that the dimensionless coefficients that multiply m_{pq} in Eqs. (2.8.2) can be used to transfer any point mass of a link pq to the same locations of point masses Y^3_{ipq} (i = 1 to 4) just by removing $\bar{k}_{pq}$ from those coefficients and replacing $\bar{x}_{pq}$ and $\bar{y}_{pq}$ with the coordinates of the point mass, as shown in Fig. 2.14.

If a link pq is grounded by a pin joint, the point mass Y^3_{1pq} located at "p" is motionless ($D^4_{1pq} = D^6_{1pq} = 0$) and should not be considered; in addition, for input torque considerations, the point masses Y^3_{2pq} and Y^3_{3pq} are also located at "p" ($D^6_{2pq} = D^6_{3pq} = 0$), and should be disregarded, too. If a ground link has constant angular velocity, further reduction of the number of linear independent D-coefficients will occur. Thus, for shaking

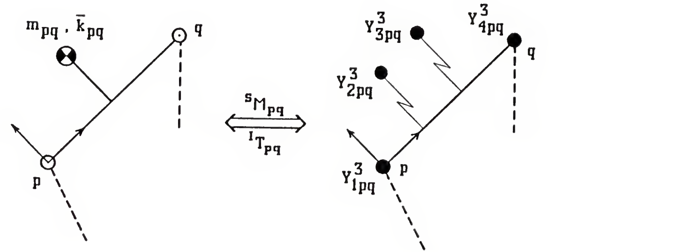


Figure 2.13 Two Link Models With Shaking Moment and Input Torque Equivalence

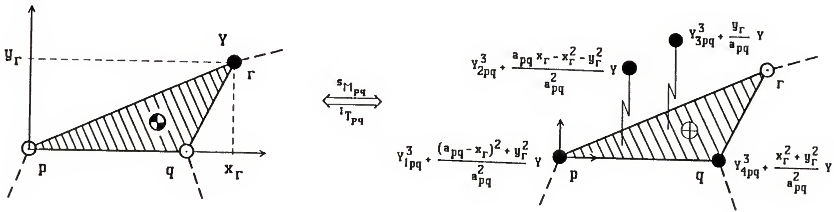


Figure 2.14 Redistribution of a Point Mass Y

moment, $D^4_{2pq} = D^4_{4pq}$, as demonstrated in Fig. 2.15; and for input torque, $D^6_{4pq} = 0$. Finally, $D^4_{2pq} = D^4_{3pq} = 0$ if the origin of the global coordinate system (the shaking moment reference point) coincides with the center of the grounded revolute joint.[1] Figure 2.16 shows the three types of grounded link configurations and how they decrease the number of linearly independent D-coefficients in the expressions for the dynamic properties of a linkage mechanism.

Figures 2.17 and 2.18 illustrate shaking moment and input torque derivations, respectively, for a four-bar linkage; and Figure 2.19 is a collection of all the basic formulations considered thus far.

Except for the slider-crank, all the mechanisms that are going to be considered in this work have only revolute joints. Hence, the contents of the last three sections will be very useful for the rest of this chapter. The expressions derived in sections 2.4, 2.5, and 2.6 for links with p-ϕ motion specification were expressed in a generic form. Therefore, with the help of Appendix C one can easily derive shaking force, shaking moment, and input torque expressions, free of linear dependencies, for linkages containing prismatic joints.

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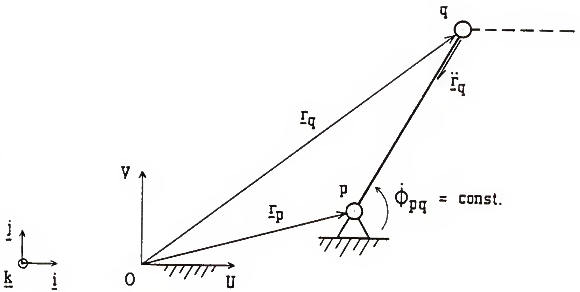
[1] Appendix B explores this fact further.

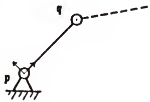
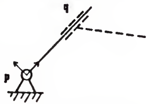
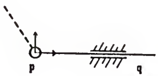


$$D^4_{4pq} = U_q \ddot{V}_q - V_q \ddot{U}_q = \underline{r}_q \times \ddot{\underline{r}}_q \cdot \underline{k} = \underline{r}_p \times \ddot{\underline{r}}_q \cdot \underline{k} =$$

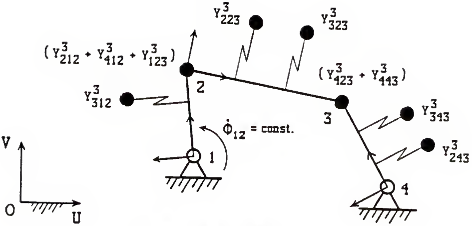
$$= U_p \ddot{V}_q - V_p \ddot{U}_q = D^4_{2pq}$$

Figure 2.15 Grounded Link With Constant Angular Velocity

Figure 2.16 Effect of Ground Links on Linear Independency of D-Coefficients

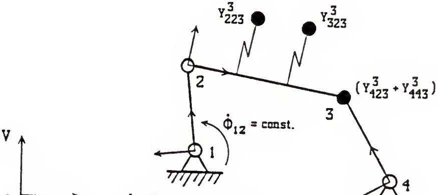
Grounded Link	Shaking Force	Shaking Moment*	Input Torque
	$D^2_{1pq} = D^2_{2pq} = 0$	$D^4_{1pq} = 0$ $D^4_{2pq} = D^4_{4pq}$ if $\dot{\Phi}_{pq} = \text{const.}$ $D^4_{2pq} = D^4_{3pq} = 0$ if "O" coincides with "p"	$D^6_{1pq} = D^6_{2pq} = D^6_{3pq} = 0$ $D^6_{4pq} = 0$ if $\dot{\Phi}_{pq} = \text{const.}$
	$D^1_{1pq} = 0$	$D^3_{1pq} = 0$ $D^3_{4pq} = 0$ if $\dot{\Phi}_{pq} = \text{const.}$ $D^3_{2pq} = D^3_{3pq} = 0$ if "O" coincides with "p"	$D^5_{1pq} = D^5_{2pq} = D^5_{3pq} = 0$ $D^5_{4pq} = 0$ if $\dot{\Phi}_{pq} = \text{const.}$
	$D^1_{2pq} = D^1_{3pq} = 0$	$D^3_{2pq} = D^3_{4pq} = 0$ $D^3_{1pq} = 0$ if "O" is along line "pq"	$D^5_{2pq} = D^5_{3pq} = D^5_{4pq} = 0$

*About the origin "O" of global coord. system



$$
\begin{aligned}
{}^{s}M &= {}^{s}M_{12} + {}^{s}M_{23} + {}^{s}M_{43} \\
&= Y^3_{312}\, D^4_{312} + (Y^3_{212} + Y^3_{412} + Y^3_{123})\, D^4_{412} + \\
&+ Y^3_{223}\, D^4_{223} + Y^3_{323}\, D^4_{323} + (Y^3_{423} + Y^3_{443})\, D^4_{443} + \\
&+ Y^3_{343}\, D^4_{343} + Y^3_{243}\, D^4_{243}
\end{aligned}
$$

Figure 2.17 Shaking Moment Derivation for a Four-Bar Linkage



$$
\begin{aligned}
{}^{I}T &= {}^{I}T_{12} + {}^{I}T_{23} + {}^{I}T_{43} \\
&= Y^3_{223}\, D^6_{223} + Y^3_{323}\, D^6_{323} + (Y^3_{423} + Y^3_{443})\, D^6_{443}
\end{aligned}
$$

Figure 2.18 Input Torque Derivation for a Four-Bar Linkage

Figure 2.19 Collection of Basic Formulations

p-Φ Motion Specification

<u>Shaking Force</u> $^{S}\underline{F}_{pq} = Y^1_{1pq}\, D^1_{1pq} +$ $+ (Y^1_{2pq} + i\, Y^1_{3pq})\, D^1_{2pq}$	$Y^1_{1pq} = m_{pq}$ $Y^1_{2pq} = m_{pq}\, \bar{x}_{pq}$ $Y^1_{3pq} = m_{pq}\, \bar{y}_{pq}$ $Y^1_{4pq} = m_{pq}(\bar{x}^2_{pq} + \bar{y}^2_{pq} + \bar{k}^2_{pq})$	$D^1_{1pq} = \ddot{U}_p + i\, \ddot{V}_p$ $D^1_{2pq} = \ddot{\Phi}_{pq}(-s_{pq} + i\, c_{pq}) - \dot{\Phi}^2_{pq}(c_{pq} + i\, s_{pq})$
<u>Shaking Moment</u> $^{S}M_{pq} = \sum_{i=1}^{4} Y^1_{ipq}\, D^3_{ipq}$		$D^3_{1pq} = U_p \ddot{V}_p - V_p \ddot{U}_p$ $D^3_{2pq} = (\ddot{V}_p c_{pq} - \ddot{U}_p s_{pq}) +$ $+ \ddot{\Phi}_{pq}(U_p c_{pq} + V_p s_{pq}) +$ $+ \dot{\Phi}^2_{pq}(-U_p s_{pq} + V_p c_{pq})$ $D^3_{3pq} = -(\ddot{V}_p s_{pq} + \ddot{U}_p c_{pq}) +$ $+ \ddot{\Phi}_{pq}(-U_p s_{pq} + V_p c_{pq}) -$ $- \dot{\Phi}^2_{pq}(U_p c_{pq} + V_p s_{pq})$ $D^3_{4pq} = \ddot{\Phi}_{pq}$
<u>Input Torque</u> $^{I}T_{pq} = \sum_{i=1}^{4} Y^1_{ipq}\, D^5_{ipq}$		$D^5_{1pq} = (\dot{U}_p \ddot{U}_p + \dot{V}_p \ddot{V}_p) \div \omega_i$ $D^5_{2pq} = [\dot{\Phi}_{pq}(-\ddot{U}_p s_{pq} + \ddot{V}_p c_{pq}) +$ $+ \ddot{\Phi}_{pq}(-\dot{U}_p s_{pq} + \dot{V}_p c_{pq}) -$ $- \dot{\Phi}^2_{pq}(\dot{U}_p c_{pq} + \dot{V}_p s_{pq})] \div \omega_i$ $D^5_{3pq} = [-\dot{\Phi}_{pq}(\ddot{U}_p c_{pq} + \ddot{V}_p s_{pq}) -$ $- \ddot{\Phi}_{pq}(\dot{U}_p c_{pq} + \dot{V}_p s_{pq}) +$ $+ \dot{\Phi}^2_{pq}(\dot{U}_p s_{pq} - \dot{V}_p c_{pq})] \div \omega_i$ $D^5_{4pq} = \dot{\Phi}_{pq}\, \ddot{\Phi}_{pq} \div \omega_i$

p-q Motion Specification

<u>Shaking Force</u> $^{S}\underline{F}_{pq} = (Y^2_{1pq} + i\, Y^2_{2pq})\, D^2_{1pq} +$ $+ (Y^2_{4pq} + i\, Y^2_{3pq})\, D^2_{4pq}$	$Y^2_{1pq} = m_{pq}(a_{pq} - \bar{x}_{pq}) \div a_{pq}$ $Y^2_{2pq} = -m_{pq}\, \bar{y}_{pq} \div a_{pq}$ $Y^2_{3pq} = m_{pq}\, \bar{y}_{pq} \div a_{pq}$ $Y^2_{4pq} = m_{pq}\, \bar{x}_{pq} \div a_{pq}$	$D^2_{1pq} = \ddot{U}_p + i\, \ddot{V}_p$ $D^2_{4pq} = \ddot{U}_q + i\, \ddot{V}_q$
<u>Shaking Moment</u> $^{S}M_{pq} = \sum_{i=1}^{4} Y^3_{ipq}\, D^4_{ipq}$	$Y^3_{1pq} = m_{pq}[(a_{pq} - \bar{x}_{pq})^2 + \bar{y}^2_{pq} + \bar{k}^2_{pq}] \div a^2_{pq}$ $Y^3_{2pq} = m_{pq}(a_{pq}\bar{x}_{pq} - \bar{x}^2_{pq} - \bar{y}^2_{pq} - \bar{k}^2_{pq}) \div a^2_{pq}$ $Y^3_{3pq} = m_{pq}\, \bar{y}_{pq} \div a_{pq}$ $Y^3_{4pq} = m_{pq}(\bar{x}^2_{pq} + \bar{y}^2_{pq} + \bar{k}^2_{pq}) \div a^2_{pq}$	$D^4_{1pq} = U_p \ddot{V}_p - V_p \ddot{U}_p$ $D^4_{2pq} = (U_p \ddot{V}_q - V_p \ddot{U}_q) + (U_q \ddot{V}_p - V_q \ddot{U}_p)$ $D^4_{3pq} = (U_p \ddot{U}_q + V_p \ddot{V}_q) - (U_q \ddot{U}_p + V_q \ddot{V}_p)$ $D^4_{4pq} = U_q \ddot{V}_q - V_q \ddot{U}_q$
<u>Input Torque</u> $^{I}T_{pq} = \sum_{i=1}^{4} Y^3_{ipq}\, D^6_{ipq}$		$D^6_{1pq} = (\dot{U}_p \ddot{U}_p + \dot{V}_p \ddot{V}_p) \div \omega_i$ $D^6_{2pq} = [(\dot{U}_p \ddot{U}_q + \dot{V}_p \ddot{V}_q) + (\dot{U}_q \ddot{U}_p + \dot{V}_q \ddot{V}_p)] \div \omega_i$ $D^6_{3pq} = [-(\dot{U}_p \ddot{V}_q - \dot{V}_p \ddot{U}_q) + (\dot{U}_q \ddot{V}_p - \dot{V}_q \ddot{U}_p)] \div \omega_i$ $D^6_{4pq} = (\dot{U}_q \ddot{U}_q + \dot{V}_q \ddot{V}_q) \div \omega_i$

2.9 The Method of Dynamic Synthesis

The synthesis method to be used in this work is the one presented in Refs. [1,35], whose description is given below.

According to the previous sections, the three dynamic properties that were considered can be expressed in the following form,

$$^{d}P = [f_1(Y's)](D)_1 + [f_2(Y's)](D)_2 + \dots + [f_n(Y's)](D)_n \tag{2.9.1}$$

where the contents of the brackets multiplying the n linearly independent D-coefficients are real-valued functions of the Y's, when either shaking moment or input torque is considered, and complex-valued functions of the Y's, if shaking force is dealt with.

For any linkage mechanism with known geometry and motion, the D-coefficients can be calculated for any position of the input link. Thus, if ^{d}P is pre-specified at n distinct values of the input angle along one cycle of operation, the following matrix equation can be formed

$$\{^{d}P\}_{nx1} = [D]_{nxn}\{f(Y's)\}_{nx1} \tag{2.9.2}$$

Since the square matrix above is generally non-singular, the result is

$$\{f(Y's)\} = [D]^{-1}\{{}^{d}P\} \tag{2.9.3}$$

When the n numerical values obtained from the right-hand-side of the above equation are substitued into Eq. (2.9.1), the synthesis of ${}^{d}P$ is accomplished. If the result is not satisfactory, the original set of synthesis specifications can be modified and the whole process repeated again.

Matrix equation (2.9.3) gives n linearly independent algebraic equations (2n for shaking force) containing the mass parameters of the linkage. They can be used as equality constraints to reduce the number of design variables when optimizing the behavior of the dynamic properties not considered in the synthesis and/or the magnitude of the reaction forces at the joints.

In Refs. [1,35], it was stated that it is always possible to concurrently satisfy both shaking force and input torque specifications. However, this assertion is not true since for some mechanisms the set of equations obtained from shaking force synthesis is not linearly independent of the set of equations obtained from input torque synthesis, as will be confirmed in the sections to follow.

2.10 Creating a Suitable Notation

In order to simplify the equality constraints represented by matrix equation (2.9.3), the lumped mass parameters below will be used for links with fixed link length:

$$x^o_{pq} = m_{pq}\bar{x}_{pq} \div a_{pq} \qquad (2.10.1)$$

$$y^o_{pq} = m_{pq}\bar{y}_{pq} \div a_{pq} \qquad (2.10.2)$$

$$k^o_{pq} = m_{pq}(\bar{x}^2_{pq}+\bar{y}^2_{pq}+\bar{k}^2_{pq}) \div a^2_{pq} \qquad (2.10.3)$$

So, from here on, the mass parameters of a link with p-q motion specification will be m_{pq}, x^o_{pq}, y^o_{pq}, and k^o_{pq}. The original mass parameters m_{pq}, $\bar{x}_{pq}$, $\bar{y}_{pq}$, and $\bar{k}_{pq}$ will be referred to as "basic mass parameters" whenever this distinction becomes necessary.

If the mass parameters defined above are substituted into Eqs. (2.7.2), the result is

$$Y^2_{1pq} = m_{pq}-x^o_{pq} \qquad (2.10.4.a)$$

$$Y^2_{2pq} = -y^o_{pq} \qquad (2.10.4.b)$$

$$Y^2_{3pq} = y^o_{pq} \qquad (2.10.4.c)$$

$$Y^2_{4pq} = x^o_{pq} \qquad (2.10.4.d)$$

and from Eqs. (2.8.2)

$$Y^3_{1pq} = m_{pq} - 2x^o_{pq} + k^o_{pq} \tag{2.10.5.a}$$

$$Y^3_{2pq} = x^o_{pq} - k^o_{pq} \tag{2.10.5.b}$$

$$Y^3_{3pq} = y^o_{pq} \tag{2.10.5.c}$$

$$Y^3_{4pq} = k^o_{pq} \tag{2.10.5.d}$$

Another simplication will be related to Eq. (2.9.1) which has been written in a general form. Thus, for a specific equation, $(D)_1$ might represent D^2_{412}, in which case $f_1(Y's)$ will turn into f^2_{412}. For example, the equation in Fig. 2.10 may be rewritten as

$$^s\underline{F} = f^2_{412} D^2_{412} + f^2_{443} D^2_{443}$$

A third and final simplification will be achieved by means of the following equalities:

$$x_{rpq} = x_r \div a_{pq}$$

$$y_{rpq} = y_r \div a_{pq}$$

where "r" is a point on link pq.

In the subsequent sections, the equality constraints which result from shaking force, shaking moment, and input torque syntheses will be derived for seven kinds of linkage mechanisms.

2.11 Equality Constraints for the Four-Bar Linkage

The functions multiplying the D-coefficients in Fig. 2.20 take the form below when Eqs. (2.10.4) and Eqs. (2.10.5) are substituted into them. They become equality constraints when their values are fixed by the synthesis equation (2.9.3).

Shaking Force:

$$\mathrm{Re}(f^2_{412}) = x^o_{12} + m_{23} - x^o_{23}$$

$$\mathrm{Im}(f^2_{412}) = y^o_{12} - y^o_{23}$$

$$\mathrm{Re}(f^2_{443}) = x^o_{43} + x^o_{23}$$

$$\mathrm{Im}(f^2_{443}) = y^o_{43} + y^o_{23}$$

Shaking Moment and Input Torque:

$$f^4_{312} = y^o_{12}$$

$$f^4_{412} = x^o_{12} + m_{23} - 2x^o_{23} + k^o_{23}$$

$$f^4_{223}, f^6_{223} = x^o_{23} - k^o_{23}$$

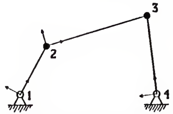
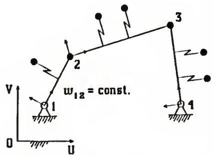
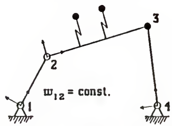
$$f^4_{323}, f^6_{323} = y^o_{23}$$

$$f^4_{443}, f^6_{443} = k^o_{23} + k^o_{43}$$

$$f^4_{343} = y^o_{43}$$

$$f^4_{243} = x^o_{43} - k^o_{43}$$

Figure 2.20 Dynamic Property Expressions for the Four-Bar Linkage

SHAKING FORCE 	$^{s}\underline{F} = [(Y^2_{412} + Y^2_{123}) + i(Y^2_{312} + Y^2_{223})]D^2_{412} + [(Y^2_{443} + Y^2_{423}) + i(Y^2_{343} + Y^2_{323})]D^2_{443}$
SHAKING MOMENT ABOUT "O" 	$^{s}M = Y^3_{312} D^4_{312} + (Y^3_{212} + Y^3_{412} + Y^3_{123}) D^4_{412} + Y^3_{223} D^4_{223} + Y^3_{323} D^4_{323}$ $+ (Y^3_{423} + Y^3_{443}) D^4_{443} + Y^3_{343} D^4_{343} + Y^3_{243} D^4_{243}$
INPUT TORQUE 	$^{I}T = Y^3_{223} D^6_{223} + Y^3_{323} D^6_{323} + (Y^3_{423} + Y^3_{443}) D^6_{443}$

It is important to notice that

$$Re(f^2_{412}) \equiv f^4_{412} + f^4_{223}$$

$$Im(f^2_{412}) \equiv f^4_{312} - f^4_{323}$$

$$Re(f^2_{443}) \equiv f^4_{223} + f^4_{443} + f^4_{243}$$

$$Im(f^2_{443}) \equiv f^4_{343} + f^4_{323}$$

Hence, for a four-bar linkage, one cannot perform shaking moment synthesis together with the synthesis of either of the other two dynamic properties. Fortunately, there is no impediment to the double syntheses of shaking force and input torque for this type of mechanism.

2.12 Equality Constraints for the Slider-Crank Mechanism

The following functions, which can also be considered as equality constraints, result from Figure 2.21.

Shaking Force:

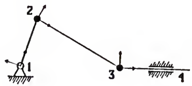
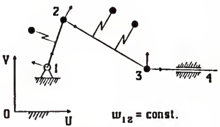
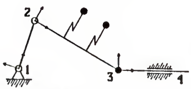
$$Re(f^2_{412}) = x^o_{12} + m_{23} - x^o_{23}$$

$$Im(f^2_{412}) = y^o_{12} - y^o_{23}$$

$$Re(f^2_{423}) = x^o_{23} + m_{34}$$

$$Im(f^2_{423}) = y^o_{23}$$

Figure 2.21 Dynamic Property Expressions for the Slider-Crank Mechanism

SHAKING FORCE 	$^{S}\underline{F} = [(Y^2_{412} + Y^2_{123}) + i(Y^2_{312} + Y^2_{223})]\,D^2_{412} + [(Y^2_{423} + Y^1_{134}) + i\,Y^2_{323}]\,D^2_{423}$
SHAKING MOMENT ABOUT "O"  w_{12} = const.	$^{S}M = Y^3_{312}\,D^4_{312} + (Y^3_{212} + Y^3_{412} + Y^3_{123})\,D^4_{412} + Y^3_{223}\,D^4_{223}$ $+ Y^3_{323}\,D^4_{323} + (Y^3_{423} + Y^1_{134})\,D^2_{423} + Y^1_{334}\,D^3_{334}$
INPUT TORQUE  w_{12} = const.	$^{I}T = Y^3_{223}\,D^6_{223} + Y^3_{323}\,D^6_{323} + (Y^3_{423} + Y^1_{134})\,D^6_{423}$

Shaking Moment and Input Torque:

$$f^4_{312} = y^o_{12}$$

$$f^4_{412} = x^o_{12} + m_{23} - 2x^o_{23} + k^o_{23}$$

$$f^4_{223}, f^6_{223} = x^o_{23} - k^o_{23}$$

$$f^4_{323}, f^6_{323} = y^o_{23}$$

$$f^4_{423}, f^6_{423} = k^o_{23} + m_{34}$$

$$f^3_{334} = m_{34}\bar{y}_{34}$$

where

$$Re(f^2_{412}) \equiv f^4_{412} + f^4_{223}$$

$$Im(f^2_{412}) \equiv f^4_{312} - f^4_{323}$$

$$Re(f^2_{423}) \equiv f^4_{223} + f^4_{423} \equiv f^6_{223} + f^6_{423}$$

$$Im(f^2_{423}) \equiv f^4_{323} \equiv f^6_{323}$$

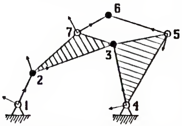
Therefore, in contrast to the four-bar linkage, no double syntheses can be carried out for the slider-crank mechanism.

2.13 Equality Constraints for the Watt I Mechanism

From Figure 2.22, the following functions (or equality constraints) can be written.

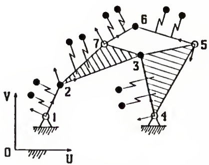
Figure 2.22 Dynamic Property Expressions for the Watt I Mechanism

SHAKING FORCE



$$^{S}\underline{F} = \{[\,Y^2_{412} + Y^2_{123} + (1-x_{723})\,Y^2_{176} + y_{723}Y^2_{276}\,] + i\,[\,Y^2_{312} + Y^2_{223} - y_{723}Y^2_{176} + (1-x_{723})\,Y^2_{276}\,]\}\,D^2_{412}$$
$$+ [\,(\,Y^2_{423} + Y^2_{443} + x_{723}Y^2_{176} - y_{723}Y^2_{276} + x_{543}Y^2_{156} - y_{543}Y^2_{256}\,)$$
$$+ i\,(\,Y^2_{323} + Y^2_{343} + x_{723}Y^2_{276} + y_{723}Y^2_{176} + x_{543}Y^2_{256} + y_{543}Y^2_{156}\,)\,]\,D^2_{443}$$
$$+ [\,(\,Y^2_{456} + Y^2_{476}\,) + i\,(\,Y^2_{356} + Y^2_{376}\,)\,]\,D^2_{456}$$

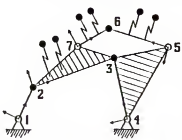
SHAKING MOMENT ABOUT "O"



$$^{S}M = Y^3_{212}D^4_{212} + Y^3_{312}D^4_{312} + \{Y^3_{412} + Y^3_{123} + [(1-x_{723})^2 + y^2_{723}]\,Y^3_{176}\}\,D^4_{412}$$
$$+ [Y^3_{223} + (x_{723} - x^2_{723} - y^2_{723})\,Y^3_{176}\,]\,D^4_{223} + (Y^3_{323} + y_{723}Y^3_{176})\,D^4_{323}$$
$$+ [\,Y^3_{423} + Y^3_{443} + (x^2_{723} + y^2_{723})\,Y^3_{176} + (x^2_{543} + y^2_{543})\,Y^3_{156}\,]\,D^4_{443}$$
$$+ [\,Y^3_{243} + (x_{543} - x^2_{543} - y^2_{543})\,Y^3_{156}\,]\,D^4_{243} + (Y^3_{343} + y_{543}Y^3_{156})\,D^4_{343}$$
$$+ Y^3_{256}D^4_{256} + Y^3_{356}D^4_{356} + (Y^3_{456} + Y^3_{476})\,D^4_{456} + Y^3_{276}D^4_{276} + Y^3_{376}D^4_{376}$$

$D^4_{212} = D^4_{412}$ if $w_{12} = \text{const.}$, and $D^4_{243} = D^4_{443}$ if $w_{43} = \text{const.}$

INPUT TORQUE



$$^{I}T = \{Y^3_{412} + Y^3_{123} + [(1-x_{723})^2 + y^2_{723}]\,Y^3_{176}\}\,D^6_{412} + [Y^3_{223} + (x_{723} - x^2_{723} - y^2_{723})\,Y^3_{176}\,]\,D^6_{223}$$
$$+ (Y^3_{323} + y_{723}Y^3_{176})\,D^6_{323} + [\,Y^3_{423} + Y^3_{443} + (x^2_{723} + y^2_{723})\,Y^3_{176} + (x^2_{543} + y^2_{543})\,Y^3_{156}\,]\,D^6_{443}$$
$$+ Y^3_{256}D^6_{256} + Y^3_{356}D^6_{356} + (Y^3_{456} + Y^3_{476})\,D^6_{456} + Y^3_{276}D^6_{276} + Y^3_{376}D^6_{376}$$

$D^6_{412} = 0$ if $w_{12} = \text{const.}$, and $D^6_{443} = 0$ if $w_{43} = \text{const.}$

$$Re(f^2_{412}) = x^o_{12}+m_{23}-x^o_{23}+(1-x_{723})m_{76}-(1-x_{723})x^o_{76}-y_{723}y^o_{76}$$

$$Im(f^2_{412}) = y^o_{12}-y^o_{23}-y_{723}m_{76}+y_{723}x^o_{76}-(1-x_{723})y^o_{76}$$

$$Re(f^2_{443}) = x^o_{23}+x^o_{43}+x_{723}m_{76}-x_{723}x^o_{76}+y_{723}y^o_{76}+x_{543}m_{56} - $$
$$- x_{543}x^o_{56}+y_{543}y^o_{56}$$

$$Im(f^2_{443}) = y^o_{23}+y^o_{43}-x_{723}y^o_{76}+y_{723}m_{76}-y_{723}x^o_{76}-x_{543}y^o_{56} +$$
$$+ y_{543}m_{56}-y_{543}x^o_{56}$$

$$Re(f^2_{456}) = x^o_{56}+x^o_{76}$$

$$Im(f^2_{456}) = y^o_{56}+y^o_{76}$$

Shaking Moment and Input Torque:

$$f^4_{212} = x^o_{12}-k^o_{12}$$

$$f^4_{312} = y^o_{12}$$

$$f^4_{412},f^6_{412} = k^o_{12}+m_{23}-2x^o_{23}+k^o_{23}+[(1-x_{723})^2+y^2_{723}]m_{76} -$$
$$- 2[(1-x_{723})^2+y^2_{723}]x^o_{76}+[(1-x_{723})^2+y^2_{723}]k^o_{76}$$

$$f^4_{223},f^6_{223} = x^o_{23}-k^o_{23}+(x_{723}-x^2_{723}-y^2_{723})m_{76}-2(x_{723} -$$
$$- x^2_{723}-y^2_{723})x^o_{76}+(x_{723}-x^2_{723}-y^2_{723})k^o_{76}$$

$$f^4_{323},f^6_{323} = y^o_{23}+y_{723}m_{76}-2y_{723}x^o_{76}+y_{723}k^o_{76}$$

$$f^4_{443}, f^6_{443} = k^o_{23} + k^o_{43} + (x^2_{723} + y^2_{723})m_{76} - 2(x^2_{723} + y^2_{723})x^o_{76} + (x^2_{723} + y^2_{723})k^o_{76} + (x^2_{543} + y^2_{543})m_{56} - 2(x^2_{543} + y^2_{543})x^o_{56} + (x^2_{543} + y^2_{543})k^o_{56}$$

$$f^4_{243} = x^o_{43} - k^o_{43} + (x_{543} - x^2_{543} - y^2_{543})m_{56} - 2(x_{543} - x^2_{543} - y^2_{543})x^o_{56} + (x_{543} - x^2_{543} - y^2_{543})k^o_{56}$$

$$f^4_{343} = y^o_{43} + y_{543}m_{56} - 2y_{543}x^o_{56} + y_{543}k^o_{56}$$

$$f^4_{256}, f^6_{256} = x^o_{56} - k^o_{56}$$

$$f^4_{356}, f^6_{356} = y^o_{56}$$

$$f^4_{456}, f^6_{456} = k^o_{56} + k^o_{76}$$

$$f^4_{276}, f^6_{276} = x^o_{76} - k^o_{76}$$

$$f^4_{376}, f^6_{376} = y^o_{76}$$

If link 12 is the input link ($D^4_{212} = D^4_{412}$ and $D^6_{412} = 0$), functions f^4_{212} and f^4_{412} must be combined, and function f^6_{412} discarded. However, if the input link is link 43 ($D^4_{243} = D^4_{443}$ and $D^6_{443} = 0$), functions f^4_{243} and f^4_{443} must be combined, instead; and f^6_{443} is the function that should be eliminated.

The following relationships exist between the previous functions:

$$Re(f^2_{412}) \equiv [f^4_{212}+f^4_{412}]+f^4_{223}+(1-x_{723})f^4_{276}-y_{723}f^4_{376}$$

$$Im(f^2_{412}) \equiv f^4_{312}-f^4_{323}-(1-x_{723})f^4_{376}-y_{723}f^4_{276}$$

$$Re(f^2_{443}) \equiv f^4_{223}+[f^4_{443}+f^4_{243}]+x_{723}f^4_{276}+x_{543}f^4_{256} - - y_{723}f^4_{376}-y_{543}f^4_{356}$$

$$Im(f^2_{443}) \equiv f^4_{323}+f^4_{343}+y_{723}f^4_{276}+y_{543}f^4_{256}-x_{723}f^4_{376} - - x_{543}f^4_{356}$$

$$Re(f^2_{456}) \equiv f^4_{256}+f^4_{456}+f^4_{276} \equiv f^6_{256}+f^6_{456}+f^6_{276}$$

$$Im(f^2_{456}) \equiv f^4_{356}+f^4_{376} \equiv f^6_{356}+f^6_{376}$$

Thus, for the Watt I linkage, no double syntheses can be performed, either.

2.14 Equality Constraints for the Watt II Mechanism

From Figure 2.23, results

Shaking Force:

$$Re(f^2_{412}) = x^o_{12}+m_{23}-x^o_{23}$$

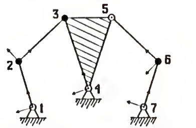
$$Im(f^2_{412}) = y^o_{12}-y^o_{23}$$

$$Re(f^2_{443}) = x^o_{23}+x^o_{43}+x_{543}x^o_{65}-y_{543}y^o_{65}$$

$$Im(f^2_{443}) = y^o_{23}+y^o_{43}+x_{543}y^o_{65}+y_{543}x^o_{65}$$

Figure 2.23 Dynamic Property Expressions for the Watt II Mechanism

SHAKING FORCE

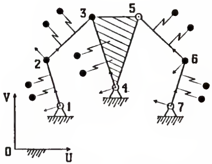


$$^{s}\underline{F} = [(Y^2_{412} + Y^2_{123}) + i(Y^2_{312} + Y^2_{223})]D^2_{412}$$

$$+[(Y^2_{423} + Y^2_{443} + x_{543}Y^2_{465} - y_{543}Y^2_{365}) + i(Y^2_{323} + Y^2_{343} + x_{543}Y^2_{365} + y_{543}Y^2_{465})]D^2_{443}$$

$$+[(Y^2_{165} + Y^2_{476}) + i(Y^2_{265} + Y^2_{376})]D^2_{476}$$

SHAKING MOMENT ABOUT "O"



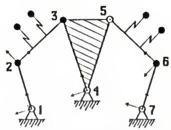
$$^{s}M = Y^3_{212}D^4_{212} + Y^3_{312}D^4_{312} + (Y^3_{412} + Y^3_{123})D^4_{412} + Y^3_{223}D^4_{223} + Y^3_{323}D^4_{323}$$

$$+[Y^3_{423} + Y^3_{443} + (x^2_{543} + y^2_{543})Y^3_{465}]D^4_{443} + [Y^3_{243} + (x_{543} - x^2_{543} - y^2_{543})Y^3_{465}]D^4_{243}$$

$$+(Y^3_{343} + y_{543}Y^3_{465})D^4_{343} + Y^3_{265}D^4_{265} + Y^3_{365}D^4_{365} + (Y^3_{165} + Y^3_{476})D^4_{476} + Y^3_{276}D^4_{276} + Y^3_{376}D^4_{376}$$

$D^4_{212} = D^4_{412}$ if $w_{12} =$ const., and $D^4_{243} = D^4_{443}$ if $w_{43} =$ const.

INPUT TORQUE



$$^{1}T = (Y^3_{412} + Y^3_{123})D^6_{412} + Y^3_{223}D^6_{223} + Y^3_{323}D^6_{323} + [Y^3_{423} + Y^3_{443} + (x^2_{543} + y^2_{543})Y^3_{465}]D^6_{443}$$

$$+ Y^3_{265}D^6_{265} + Y^3_{365}D^6_{365} + (Y^3_{165} + Y^3_{476})D^6_{476}$$

$D^6_{412} = 0$ if $w_{12} =$ const., and $D^6_{443} = 0$ if $w_{43} =$ const.

$$Re(f^{2}_{476}) = m_{65}-x^{o}_{65}+x^{o}_{76}$$

$$Im(f^{2}_{476}) = -y^{o}_{65}-y^{o}_{76}$$

<u>Shaking Moment and Input Torque</u>:

$$f^{4}_{212} = x^{o}_{12}-k^{o}_{12}$$

$$f^{4}_{312} = y^{o}_{12}$$

$$f^{4}_{412}, f^{6}_{412} = k^{o}_{12}+m_{23}-2x^{o}_{23}+k^{o}_{23}$$

$$f^{4}_{223}, f^{6}_{223} = x^{o}_{23}-k^{o}_{23}$$

$$f^{4}_{323}, f^{6}_{323} = y^{o}_{23}$$

$$f^{4}_{443}, f^{6}_{443} = k^{o}_{23}+k^{o}_{43}+(x^{2}_{543}+y^{2}_{543})k^{o}_{65}$$

$$f^{4}_{243} = x^{o}_{43}-k^{o}_{43}+(x_{543}-x^{2}_{543}-y^{2}_{543})k^{o}_{65}$$

$$f^{4}_{343} = y^{o}_{43}+y_{543}k^{o}_{65}$$

$$f^{4}_{265}, f^{6}_{265} = x^{o}_{65}-k^{o}_{65}$$

$$f^{4}_{365}, f^{6}_{365} = y^{o}_{65}$$

$$f^{4}_{476}, f^{6}_{476} = m_{65}-2x^{o}_{65}+k^{o}_{65}+k^{o}_{76}$$

$$f^{4}_{276} = x^{o}_{76}-k^{o}_{76}$$

$$f^{4}_{376} = y^{o}_{76}$$

If ω_{12} = const. $\rightarrow f^{4}_{212}+f^{4}_{412} = f^{4}_{412new}$; and f^{6}_{412} must be

discarded. If ω_{43} = const. $\rightarrow$ $f^4_{243}+f^4_{443} = f^4_{443new}$; and f^6_{443} must be discarded.

The following relationships can be written from above:

$$Re(f^2_{412}) \equiv [f^4_{212}+f^4_{412}]+f^4_{223}$$

$$Im(f^2_{412}) \equiv f^4_{312}-f^4_{323}$$

$$Re(f^2_{443}) \equiv f^4_{223}+[f^4_{443}+f^4_{243}]+x_{543}f^4_{265}-y_{543}f^4_{365}$$

$$Im(f^2_{443}) \equiv f^4_{323}+f^4_{343}+y_{543}f^4_{265}+x_{543}f^4_{365}$$

$$Re(f^2_{476}) \equiv f^4_{476}+f^4_{265}+f^4_{276}$$

$$Im(f^2_{476}) \equiv f^4_{376}-f^4_{365}$$

Therefore, the double syntheses of shaking force and input torque can be performed for the Watt II mechanism.

2.15 Equality Constraints for the Stephenson I Mechanism

The following equalities result from Figure 2.24.

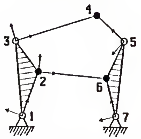
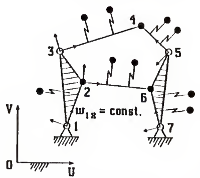
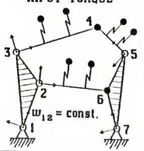
Shaking Force:

$$Re(f^2_{412}) = x^o_{12}+m_{26}-x^o_{26}+x_{312}m_{34}-x_{312}x^o_{34}+y_{312}y^o_{34}$$

$$Im(f^2_{412}) = y^o_{12}-y^o_{26}-x_{312}y^o_{34}+y_{312}m_{34}-y_{312}x^o_{34}$$

$$Re(f^2_{476}) = x^o_{26}+x^o_{76}+x_{576}m_{54}-x_{576}x^o_{54}+y_{576}y^o_{54}$$

Figure 2.24 Dynamic Property Expressions for the Stephenson I Mechanism

SHAKING FORCE 	$^{s}\underline{F} = [(Y^2_{412} + Y^2_{126} + x_{312}Y^2_{134} - y_{312}Y^2_{234}) + i(Y^2_{312} + Y^2_{226} + x_{312}Y^2_{234} + y_{312}Y^2_{134})]D^2_{412}$ $+ [(Y^2_{426} + Y^2_{476} + x_{576}Y^2_{154} - y_{576}Y^2_{254}) + i(Y^2_{326} + Y^2_{376} + x_{576}Y^2_{254} + y_{576}Y^2_{154})]D^2_{476}$ $+ [(Y^2_{434} + Y^2_{454}) + i(Y^2_{334} + Y^2_{354})]D^2_{434}$
SHAKING MOMENT ABOUT "O" 	$^{s}M = (Y^3_{312} + y_{312}Y^3_{134})D^4_{312} + [Y^3_{212} + (x_{312} - x^2_{312} - y^2_{312})Y^3_{134} + Y^3_{412} + Y^3_{126} + (x^2_{312} + y^2_{312})Y^3_{134}]D^4_{412}$ $+ Y^3_{226}D^4_{226} + Y^3_{326}D^4_{326} + [Y^3_{426} + Y^3_{476} + (x^2_{576} + y^2_{576})Y^3_{154}]D^4_{476}$ $+ [Y^3_{276} + (x_{576} - x^2_{576} - y^2_{576})Y^3_{154}]D^4_{276} + (Y^3_{376} + y_{576}Y^3_{154})D^4_{376} + Y^3_{234}D^4_{234} + Y^3_{334}D^4_{334}$ $+ (Y^3_{434} + Y^3_{454})D^4_{434} + Y^3_{254}D^4_{254} + Y^3_{354}D^4_{354}$
INPUT TORQUE 	$^{I}T = Y^3_{226}D^6_{226} + Y^3_{326}D^6_{326} + [Y^3_{426} + Y^3_{476} + (x^2_{576} + y^2_{576})Y^3_{154}]D^6_{476} + Y^3_{234}D^6_{234} + Y^3_{334}D^6_{334}$ $+ (Y^3_{434} + Y^3_{454})D^6_{434} + Y^3_{254}D^6_{254} + Y^3_{354}D^6_{354}$

$$\mathrm{Im}(f^{2}_{476}) = y^{o}_{26}+y^{o}_{76}-x_{576}y^{o}_{54}+y_{576}m_{54}-y_{576}x^{o}_{54}$$

$$\mathrm{Re}(f^{2}_{434}) = x^{o}_{34}+x^{o}_{54}$$

$$\mathrm{Im}(f^{2}_{434}) = y^{o}_{34}+y^{o}_{54}$$

<u>Shaking Moment and Input Torque:</u>

$$f^{4}_{312} = y^{o}_{12}+y_{312}m_{34}-2y_{312}x^{o}_{34}+y_{312}k^{o}_{34}$$

$$f^{4}_{412} = x^{o}_{12}+x_{312}m_{34}-2x_{312}x^{o}_{34}+x_{312}k^{o}_{34}+m_{26}-2x^{o}_{26}+k^{o}_{26}$$

$$f^{4}_{226},f^{6}_{226} = x^{o}_{26}-k^{o}_{26}$$

$$f^{4}_{326},f^{6}_{326} = y^{o}_{26}$$

$$f^{4}_{476},f^{6}_{476} = k^{o}_{26}+k^{o}_{76}+(x^{2}_{576}+y^{2}_{576})m_{54}-2(x^{2}_{576}+y^{2}_{576})x^{o}_{54} + (x^{2}_{576}+y^{2}_{576})k^{o}_{54}$$

$$f^{4}_{276} = x^{o}_{76}-k^{o}_{76}+(x_{576}-x^{2}_{576}-y^{2}_{576})m_{54}-2(x_{576}-x^{2}_{576} - y^{2}_{576})x^{o}_{54}+(x_{576}-x^{2}_{576}-y^{2}_{576})k^{o}_{54}$$

$$f^{4}_{376} = y^{o}_{76}+y_{576}m_{54}-2y_{576}x^{o}_{54}+y_{576}k^{o}_{54}$$

$$f^{4}_{234},f^{6}_{234} = x^{o}_{34}-k^{o}_{34}$$

$$f^{4}_{334},f^{6}_{334} = y^{o}_{34}$$

$$f^{4}_{434},f^{6}_{434} = k^{o}_{34}+k^{o}_{54}$$

$$f^{4}_{254},f^{6}_{254} = x^{o}_{54}-k^{o}_{54}$$

$f^4_{354}, f^6_{354} = y^o_{54}$

The above functions are related in the following way:

$$\mathrm{Re}(f^2_{412}) \equiv f^4_{412} + f^4_{226} + x_{312} f^4_{234} + y_{312} f^4_{334}$$

$$\mathrm{Im}(f^2_{412}) \equiv f^4_{312} + y_{312} f^4_{234} - x_{312} f^4_{334} - f^4_{326}$$

$$\mathrm{Re}(f^2_{476}) \equiv f^4_{226} + f^4_{276} + f^4_{476} + x_{576} f^4_{254} + y_{576} f^4_{354}$$

$$\mathrm{Im}(f^2_{476}) \equiv f^4_{326} + f^4_{376} + y_{576} f^4_{254} - x_{576} f^4_{354}$$

$$\mathrm{Re}(f^2_{434}) \equiv f^4_{234} + f^4_{434} + f^4_{254} \equiv f^6_{234} + f^6_{434} + f^6_{254}$$

$$\mathrm{Im}(f^2_{434}) \equiv f^4_{334} + f^4_{354} \equiv f^6_{334} + f^6_{354}$$

Hence, no double syntheses can be executed for the Stephenson I mechanism.

2.16 Equality Constraints for the Stephenson II Mechanism

The following functions can be written from Figure 2.25.

Shaking Force:

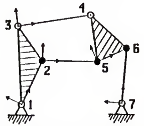
$$\mathrm{Re}(f^2_{412}) = x^o_{12} + m_{25} - x^o_{25} + x_{312} m_{34} - x_{312} x^o_{34} + y_{312} y^o_{34}$$

$$\mathrm{Im}(f^2_{412}) = y^o_{12} - y^o_{25} - x_{312} y^o_{34} + y_{312} m_{34} - y_{312} x^o_{34}$$

$$\mathrm{Re}(f^2_{156}) = x^o_{25} + m_{56} - x^o_{56} + (1 - x_{456}) x^o_{34} + y_{456} y^o_{34}$$

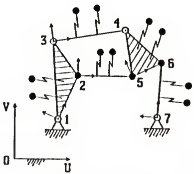
Figure 2.25 Dynamic Property Expressions for the Stephenson II Mechanism

SHAKING FORCE



$$
{}^{S}\underline{F} = [\,(Y^2_{412} + Y^2_{125} + x_{312}Y^2_{134} - y_{312}Y^2_{234}) + i\,(Y^2_{312} + Y^2_{225} + x_{312}Y^2_{234} + y_{312}Y^2_{134})\,]\,D^2_{412}
$$
$$
+ \{\,[\,Y^2_{425} + Y^2_{156} + (1 - x_{456})Y^2_{434} + y_{456}Y^2_{334}\,] + i\,[\,Y^2_{326} + Y^2_{256} + (1 - x_{456})Y^2_{334} - y_{456}Y^2_{434}\,]\,\}\,D^2_{156}
$$
$$
+ [\,(Y^2_{456} + Y^2_{476} + x_{456}Y^2_{434} - y_{456}Y^2_{334}) + i\,(Y^2_{356} + Y^2_{376} + x_{456}Y^2_{334} + y_{456}Y^2_{434})\,]\,D^2_{476}
$$

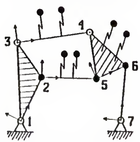
SHAKING MOMENT ABOUT "O"



$$
{}^{S}M = [\,Y^3_{212} + (x_{312} - x^2_{312} - y^2_{312})Y^3_{134}\,]\,D^4_{212} + (Y^3_{312} + y_{312}Y^3_{134})\,D^4_{312}
$$
$$
+ [\,Y^3_{412} + Y^3_{125} + (x^2_{312} + y^2_{312})Y^3_{134}\,]\,D^4_{412} + Y^3_{225}D^4_{225} + Y^3_{325}D^4_{325}
$$
$$
+ \{\,Y^3_{425} + Y^3_{156} + [\,(1 - x_{456})^2 + y^2_{456}\,]\,Y^3_{434}\,\}\,D^4_{156}
$$
$$
+ [\,Y^3_{256} + (x_{456} - x^2_{456} - y^2_{456})Y^3_{434}\,]\,D^4_{256} + (Y^3_{356} + y_{456}Y^3_{434})\,D^4_{356}
$$
$$
+ [\,Y^3_{456} + Y^3_{476} + (x^2_{456} + y^2_{456})Y^3_{434}\,]\,D^4_{476} + Y^3_{276}D^4_{276} + Y^3_{376}D^4_{376} + Y^3_{234}D^4_{234} + Y^3_{334}D^4_{334}
$$

$D^4_{212} = D^4_{412}$ if $\omega_{12} = \text{const.}$, and $D^4_{276} = D^4_{476}$ if $\omega_{76} = \text{const.}$

INPUT TORQUE



$$
{}^{I}T = [\,Y^3_{412} + Y^3_{125} + (x^2_{312} + y^2_{312})Y^3_{134}\,]\,D^6_{412} + Y^3_{225}D^6_{225} + Y^3_{325}D^6_{325}
$$
$$
+ \{\,Y^3_{425} + Y^3_{156} + [\,(1 - x_{456})^2 + y^2_{456}\,]\,Y^3_{434}\,\}\,D^6_{156} + [\,Y^3_{256} + (x_{456} - x^2_{456} - y^2_{456})Y^3_{434}\,]\,D^6_{256}
$$
$$
+ (Y^3_{356} + y_{456}Y^3_{434})\,D^6_{356} + [\,Y^3_{456} + Y^3_{476} + (x^2_{456} + y^2_{456})Y^3_{434}\,]\,D^6_{476} + Y^3_{234}D^6_{234} + Y^3_{334}D^6_{334}.
$$

$D^6_{412} = 0$ if $\omega_{12} = \text{const.}$, and $D^6_{476} = 0$ if $\omega_{76} = \text{const.}$

$$Im(f^{2}_{156}) = y^{o}_{25}-y^{o}_{56}+(1-x_{456})y^{o}_{34}-y_{456}x^{o}_{34}$$

$$Re(f^{2}_{476}) = x^{o}_{56}+x^{o}_{76}+x_{456}x^{o}_{34}-y_{456}y^{o}_{34}$$

$$Im(f^{2}_{476}) = y^{o}_{56}+y^{o}_{76}+x_{456}y^{o}_{34}+y_{456}x^{o}_{34}$$

Shaking Moment and Input Torque:

$$f^{4}_{212} = x^{o}_{12}-k^{o}_{12}+(x_{312}-x^{2}_{312}-y^{2}_{312})m_{34}-2(x_{312}-x^{2}_{312} - y^{2}_{312})x^{o}_{34}+(x_{312}-x^{2}_{312}-y^{2}_{312})k^{o}_{34}$$

$$f^{4}_{312} = y^{o}_{12}+y_{312}m_{34}-2y_{312}x^{o}_{34}+y_{312}k^{o}_{34}$$

$$f^{4}_{412},f^{6}_{412} = k^{o}_{12}+m_{25}-2x^{o}_{25}+k^{o}_{25}+(x^{2}_{312}+y^{2}_{312})m_{34}-2(x^{2}_{312} + y^{2}_{312})x^{o}_{34}+(x^{2}_{312}+y^{2}_{312})k^{o}_{34}$$

$$f^{4}_{225},f^{6}_{225} = x^{o}_{25}-k^{o}_{25}$$

$$f^{4}_{325},f^{6}_{325} = y^{o}_{25}$$

$$f^{4}_{156},f^{6}_{156} = k^{o}_{25}+m_{56}-2x^{o}_{56}+k^{o}_{56}+[(1-x_{456})^{2}+y^{2}_{456}]k^{o}_{34}$$

$$f^{4}_{256},f^{6}_{256} = x^{o}_{56}-k^{o}_{56}+(x_{456}-x^{2}_{456}-y^{2}_{456})k^{o}_{34}$$

$$f^{4}_{356},f^{6}_{356} = y^{o}_{56}+y_{456}k^{o}_{34}$$

$$f^{4}_{476},f^{6}_{476} = k^{o}_{56}+k^{o}_{76}+(x^{2}_{456}+y^{2}_{456})k^{o}_{34}$$

$$f^{4}_{276} = x^{o}_{76}-k^{o}_{76}$$

$$f^{4}_{376} = y^{o}_{76}$$

$f^4_{234}, f^6_{234} = x^o_{34} - k^o_{34}$

$f^4_{334}, f^6_{334} = y^o_{34}$

If ω_{12} = const. $\rightarrow f^4_{212} + f^4_{412} = f^4_{412new}$; and f^6_{412} must be discarded. If ω_{76} = const. $\rightarrow f^4_{276} + f^4_{476} = f^4_{476new}$; and f^6_{476} must be discarded.

The following relationships exist between the previous functions:

$$\mathrm{Re}(f^2_{412}) \equiv [f^4_{212} + f^4_{412}] + f^4_{225} + x_{312} f^4_{234} + y_{312} f^4_{334}$$

$$\mathrm{Im}(f^2_{412}) \equiv f^4_{312} - f^4_{325} + y_{312} f^4_{234} - x_{312} f^4_{334}$$

$$\mathrm{Re}(f^2_{156}) \equiv f^4_{225} + f^4_{156} + f^4_{256} + (1 - x_{456}) f^4_{234} + y_{456} f^4_{334}$$

$$\equiv f^6_{225} + f^6_{156} + f^6_{256} + (1 - x_{456}) f^6_{234} + y_{456} f^6_{334}$$

$$\mathrm{Im}(f^2_{156}) \equiv f^4_{325} - f^4_{356} - y_{456} f^4_{234} + (1 - x_{456}) f^4_{334}$$

$$\equiv f^6_{325} - f^6_{356} - y_{456} f^6_{234} + (1 - x_{456}) f^6_{334}$$

$$\mathrm{Re}(f^2_{476}) \equiv f^4_{256} + [f^4_{476} + f^4_{276}] + x_{456} f^4_{234} - y_{456} f^4_{334}$$

$$\mathrm{Im}(f^2_{476}) \equiv f^4_{356} + f^4_{376} + y_{456} f^4_{234} + x_{456} f^4_{334}$$

Thus, no double syntheses can be performed for the Stephenson II mechanism, either.

2.17 Equality Constraints for the Stephenson III Mechanism

Figure 2.26 yields the following equalities.

Shaking Force:

$$\mathrm{Re}(f^2_{412}) = x^o_{12}+m_{23}-x^o_{23}+(1-x_{523})m_{56}-(1-x_{523})x^o_{56}-y_{523}y^o_{56}$$

$$\mathrm{Im}(f^2_{412}) = y^o_{12}-y^o_{23}-(1-x_{523})y^o_{56}-y_{523}m_{56}+y_{523}x^o_{56}$$

$$\mathrm{Re}(f^2_{443}) = x^o_{23}+x^o_{43}+x_{543}m_{56}-x_{543}x^o_{56}+y_{543}y^o_{56}$$

$$\mathrm{Im}(f^2_{443}) = y^o_{23}+y^o_{43}-x_{543}y^o_{56}+y_{543}m_{56}-y_{543}x^o_{56}$$

$$\mathrm{Re}(f^2_{476}) = x^o_{56}+x^o_{76}$$

$$\mathrm{Im}(f^2_{476}) = y^o_{56}+y^o_{76}$$

Shaking Moment and Input Torque:

$$f^4_{212} = x^o_{12}-k^o_{12}$$

$$f^4_{312} = y^o_{12}$$

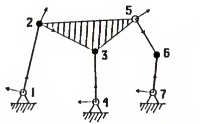
$$f^4_{412},f^6_{412} = k^o_{12}+m_{23}-2x^o_{23}+k^o_{23}+[(1-x_{523})^2+y^2_{523}]m_{56}-$$
$$-2[(1-x_{523})^2+y^2_{523}]x^o_{56}+[(1-x_{523})^2+y^2_{523}]k^o_{56}$$

$$f^4_{223},f^6_{223} = x^o_{23}-k^o_{23}+(x_{523}-x^2_{523}-y^2_{523})m_{56}-2(x_{523}-x^2_{523}-$$
$$-y^2_{523})x^o_{56}+(x_{523}-x^2_{523}-y^2_{523})k^o_{56}$$

$$f^4_{323},f^6_{323} = y^o_{23}+y_{523}m_{56}-2y_{523}x^o_{56}+y_{523}k^o_{56}$$

Figure 2.26 Dynamic Property Expressions for the Stephenson III Mechanism

SHAKING FORCE

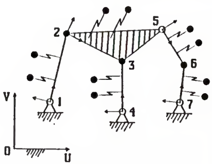


$$^{s}\underline{F} = \{ [Y^2_{412} + Y^2_{123} + (1 - x_{523}) Y^2_{156} + y_{523} Y^2_{256}] + i [Y^2_{312} + Y^2_{223} + (1 - x_{623}) Y^2_{256} - y_{523} Y^2_{156}] \} D^2_{412}$$

$$+ [(Y^2_{423} + Y^2_{443} + x_{543} Y^2_{156} - y_{543} Y^2_{256}) + i (Y^2_{323} + Y^2_{343} + x_{543} Y^2_{256} + y_{543} Y^2_{156})] D^2_{443}$$

$$+ [(Y^2_{456} + Y^2_{476}) + i (Y^2_{356} + Y^2_{376})] D^2_{476}$$

SHAKING MOMENT ABOUT "O"



$$^{s}M = Y^3_{212} D^4_{212} + Y^3_{312} D^4_{312} + \{ Y^3_{412} + Y^3_{123} + [(1 - x_{623})^2 + y^2_{523}] Y^3_{156} \} D^4_{412}$$

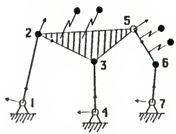
$$+ [Y^3_{223} + (x_{523} - x^2_{623} - y^2_{623}) Y^3_{156}] D^4_{223} + (Y^3_{323} + y_{523} Y^3_{156}) D^4_{323}$$

$$+ [Y^3_{423} + Y^3_{443} + (x^2_{643} + y^2_{643}) Y^3_{156}] D^4_{443} + Y^3_{243} D^4_{243} + Y^3_{343} D^4_{343}$$

$$+ Y^3_{256} D^4_{256} + Y^3_{356} D^4_{356} + (Y^3_{456} + Y^3_{476}) D^4_{476} + Y^3_{276} D^4_{276} + Y^3_{376} D^4_{376}$$

$D^4_{212} = D^4_{412}$ if ω_{12} = const., and $D^4_{276} = D^4_{476}$ if ω_{76} = const.

INPUT TORQUE



$$^{I}T = \{ Y^3_{412} + Y^3_{123} + [(1 - x_{523})^2 + y^2_{523}] Y^3_{156} \} D^6_{412} + [Y^3_{223} + (x_{523} - x^2_{523} - y^2_{523}) Y^3_{156}] D^6_{223}$$

$$+ (Y^3_{323} + y_{523} Y^3_{156}) D^6_{323} + [Y^3_{423} + Y^3_{443} + (x^2_{543} + y^2_{543}) Y^3_{156}] D^6_{443}$$

$$+ Y^3_{256} D^6_{256} + Y^3_{356} D^6_{356} + (Y^3_{456} + Y^3_{476}) D^6_{476}$$

$D^6_{412} = 0$ if ω_{12} = const., and $D^6_{476} = 0$ if ω_{76} = const.

$$f^4_{443}, f^6_{443} = k^o_{23} + k^o_{43} + (x^2_{543} + y^2_{543})m_{56} - 2(x^2_{543} + y^2_{543})x^o_{56} + (x^2_{543} + y^2_{543})k^o_{56}$$

$$f^4_{243} = x^o_{43} - k^o_{43}$$

$$f^4_{343} = y^o_{43}$$

$$f^4_{256}, f^6_{256} = x^o_{56} - k^o_{56}$$

$$f^4_{356}, f^6_{356} = y^o_{56}$$

$$f^4_{476}, f^6_{476} = k^o_{56} + k^o_{76}$$

$$f^4_{276} = x^o_{76} - k^o_{76}$$

$$f^4_{376} = y^o_{76}$$

If ω_{12} = const. $\rightarrow f^4_{212} + f^4_{412} = f^4_{412\text{new}}$; and f^6_{412} must be discarded. If ω_{76} = const. $\rightarrow f^4_{276} + f^4_{476} = f^4_{476\text{new}}$; and f^6_{476} must be discarded.

From above, the following relationships may be deduced.

$$\mathrm{Re}(f^2_{412}) \equiv [f^4_{212} + f^4_{412}] + f^4_{223} + (1 - x_{523})f^4_{256} - y_{523}f^4_{356}$$

$$\mathrm{Im}(f^2_{412}) \equiv f^4_{312} - f^4_{323} - y_{523}f^4_{256} - (1 - x_{523})f^4_{356}$$

$$\mathrm{Re}(f^2_{443}) \equiv f^4_{223} + f^4_{443} + f^4_{243} + x_{523}f^4_{256} + y_{523}f^4_{356}$$

$$\mathrm{Im}(f^2_{443}) \equiv f^4_{323} + f^4_{343} - x_{523}f^4_{356} + y_{523}f^4_{256}$$

$$Re(f^2_{476}) \equiv f^4_{256}+f^4_{476}+f^4_{276}$$

$$Im(f^2_{476}) \equiv f^4_{356}+f^4_{376}$$

Hence, it is feasible to carry out the double syntheses of shaking force and input torque for the Stephenson III mechanism.

2.18 Comments

It is important to notice from the previous sections that the masses of the links grounded by a revolute joint never appeared explicitly in the equality constraints. This was achieved because the origin of the local coordinate system of each one of those links was attached to the center of the grounded pin joint, in accordance with "Rule 1" of section 2.2. As a reinforcment to the importance of this fact, the masses of links grounded by pin joints do not appear explicitly in the derivations for the reaction forces either, as demonstrated in Appendix D.

All the sets of equality constraints obtained previously contain more variables than equations, hence some of the variables need to be pre-specified in order to solve for the others. Fortunately, the remaining variables can always be calculated by judiciously solving the equations sequentially, even for the cases of double syntheses.

Table 2.1 presents some key information, from the last seven sections, relevant to the cases of dynamic synthesis that convey fewer free mass parameters to the optimization process that will be described in the next chapter.

Table 2.1
Cases of Dynamic Synthesis Leaving Fewer Free Mass Parameters for Optimization

Mechanism	Synthesis Specification	Mass Parameters in Equality Constraints	Possible Mass Parameters for Optimization	Mass Parameters With No Individual Influence
Four-Bar	$\frac{7\ {}^{S}M}{2\ {}^{S}\underline{F} + 3\ {}^{I}T}$	9	m_{23} and x^{o}_{23}	m_{12}, m_{43} and k^{o}_{12}
Slider Crank	$6\ {}^{S}M$	8	m_{23} and x^{o}_{23}	$m_{12}, k^{o}_{12}, x^{o}_{34}$ and k^{o}_{34}
Watt I	$12\ {}^{S}M$	17	$m_{23}, x^{o}_{23}, m_{56}, m_{76}$ and x^{o}_{76}	m_{12}, m_{43} and k^{o}_{12}(or k^{o}_{43})[a]
Watt II	$\frac{12\ {}^{S}M}{3\ {}^{S}\underline{F} + 6\ {}^{I}T}$	16	$m_{23}, x^{o}_{23}, m_{65}$ and x^{o}_{65}	m_{12}, m_{43}, m_{76} and k^{o}_{12}(or k^{o}_{43})[a]
Stephenson I	$12\ {}^{S}M$	17	$m_{26}, x^{o}_{26}, m_{34}, x^{o}_{34}$ and m_{54}	m_{12}, m_{76} and k^{o}_{12}
Stephenson II	$12\ {}^{S}M$	17	$m_{25}, x^{o}_{25}, m_{34}, x^{o}_{34}$ and x^{o}_{56}	m_{12}, m_{76} and k^{o}_{12}(or k^{o}_{76})[b]
Stephenson III	$\frac{12\ {}^{S}M}{3\ {}^{S}\underline{F} + 6\ {}^{I}T}$	16	$m_{23}, x^{o}_{23}, m_{56}$ and x^{o}_{56}	m_{12}, m_{43}, m_{76} and k^{o}_{12} (or k^{o}_{76})[b]

[a] If link 43 is the input link; [b] If link 76 is the input link.

CHAPTER III
DYNAMIC OPTIMIZATION

In this chapter, the main aspects of a proposed interactive software package for the dynamic optimization of linkage mechanisms will be considered. A package based on the contents of this chapter has actually been created, and will be described in the next chapter.

3.1 Optimization in Mechanism Design

The development of the digital computer brought a new impetus to the general field of optimization. Previously infeasible approaches to optimization became practical giving new directions to innumerous areas of human knowledge, in particular the area of mechanism design.

Some of the earliest works that applied the digital computer on linkage optimization problems were performed by Freudenstein and Sandor [74,75] in 1959. They synthesized path generating mechanisms by means of complex number theory and an IBM 650 computer. Since then, the number of contributions to mechanism design optimization has increased exponentially. Undoubtedly, most of these contributions have emphasized the iterative methods [76-88],

i.e., methods that involve "mathematical programming" techniques.

As opposed to iterative methods, random search methods look for the best solution by exhaustively considering the points of a grid which are representative of the whole design space. Although the random search[1] methods are in general computationally more expensive than the iterative ones, there are some techniques that can increase their efficiency considerably.

An early use of the random search concept in linkage optimization occurred in 1962 when Roth, Sandor and Freudenstein [89] worked on the synthesis of four-bar path generating mechanisms with optimum transmissions characteristics.

In 1969, Eschenbach and Tesar [90] introduced the technique of "sequential filtering" by means of a random search method for the optimal design of crank and rocker four-bars. Essentially, a large set of linkages satisfying four prescribed multiply separated positions and certain necessary conditions was generated computationally and ranked according to numerous desirable conditions acting as sequential filters. Later, Spitznagel [91], and

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1 In this work, the term "random search" has a broad sense, that is, a random search method does not necessarily select the design points randomly.

Spitznagel and Tesar [92] improved this approach by introducing the idea of "grid expansion." Through this technique, a region on the grid about the linkages of higher quality is selected, then a more refined grid is generated about the selected region.

The philosophies of "sequential filtering" and "grid expansion" have been implemented in the interactive software package that will be described in Chapter IV.

3.2 Definition of the Problem

Given a set of N linkages with unknown mass parameters, possibly coming from a previous kinematic optimization process, it is desirable to find by means of an interactive software package the best distribution of internal masses for each of these linkages in order to optimize their dynamic behavior. Then, from the N "optimum" solutions, select the linkage that has the best performance with respect to all the criteria of interest to the designer. It is also desirable to be able to balance a pre-existing linkage with known mass parameters.

To solve such a problem, the combination of synthesis and optimization will be considered next.

3.3 Integration of Synthesis and Optimization

The flowchart in Fig. 3.1 shows the primary elements necessary to the integration of dynamic synthesis and

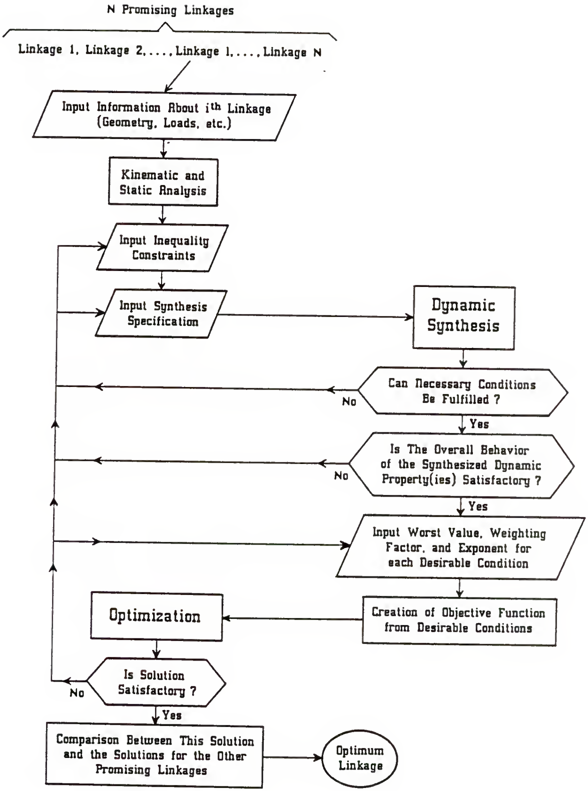


Figure 3.1 Integration of Dynamic Synthesis and Optimization in Computer Software

optimization in an interactive software package applicable to the problem described in the preceding section.

The dynamic synthesis process considered in Chapter II represents the first stage of the whole optimization method. As explained in section 2.9, the equality constraints obtained from synthesis decrease the number of free design variables which are carried over to the main optimization stage. This fact was summarized in Table 2.1 for ten cases of dynamic synthesis. However, as indicated in Fig. 3.1, synthesis can be considered successful only if the necessary conditions can be fulfilled and the overall behavior of the dynamic property(ies) being synthesized is considered satisfactory.

In the remaining part of this chapter, the parts of the chart in Fig. 3.1 needing elucidation will be considered.

3.4 The Necessary Conditions

A mechanism undergoing dynamic optimization must have its mass parameters subjected to side constraints. In this work, these constraints are represented by the inequalities,

$$m_{pqmin} \leq m_{pq} \leq m_{pqmax} \tag{3.4.1.a}$$

$$x_{pqmin} \leq x_{pq} \leq x_{pqmax} \tag{3.4.1.b}$$

$$y_{pqmin} \leq y_{pq} \leq y_{pqmax} \tag{3.4.1.c}$$

$$k_{pqmin} \leq k_{pq} \leq k_{pqmax} \qquad (3.4.1.d)$$

where m_{pq}, x_{pq}, y_{pq}, and k_{pq} are the basic mass parameters of a generic link pq representing all the moving links of such a mechanism. Thus, using the definition of the (lumped) mass parameters given in section 2.10, it is possible to transform the constraints above into the following inequalities:

$$m_{pqmin} \leq m_{pq} \leq m_{pqmax} \qquad (3.4.2.a)$$

$$x^o_{pqmin} \leq x^o_{pq} \leq x^o_{pqmax} \qquad (3.4.2.b)$$

$$y^o_{pqmin} \leq y^o_{pq} \leq y^o_{pqmax} \qquad (3.4.2.c)$$

$$k^o_{pqmin} \leq k^o_{pq} \leq k^o_{pqmax} \qquad (3.4.2.d)$$

where

$$x^o_{pqmin} = m_{pqmax} \cdot x_{pqmin} \div a_{pq} \quad \text{if } x_{pqmin} \leq 0$$

$$\text{or } = m_{pqmin} \cdot x_{pqmin} \div a_{pq} \quad \text{if } x_{pqmin} \geq 0$$

$$x^o_{pqmax} = m_{pqmax} \cdot x_{pqmax} \div a_{pq} \quad \text{if } x_{pqmax} \geq 0$$

$$\text{or } = m_{pqmin} \cdot x_{pqmax} \div a_{pq} \quad \text{if } x_{pqmax} \leq 0$$

(y^o_{pqmin} and y^o_{pqmax} are calculated similarly)

$$k^o_{pqmin} = m_{pqmin}\{[\mathrm{MIN}(|x_{pqmin}|,|x_{pqmax}|)]^2 + \\ + [\mathrm{MIN}(|y_{pqmin}|,|y_{pqmax}|)]^2 + k^2_{pqmin}\} \div a^2_{pq}$$

and $$k^o_{pqmax} = m_{pqmax}\{[MAX(|x_{pqmin}|,|x_{pqmax}|)]^2 + [MAX(|y_{pqmin}|,|y_{pqmax}|)]^2 + k^2_{pqmax}\} \div a^2_{pq}$$

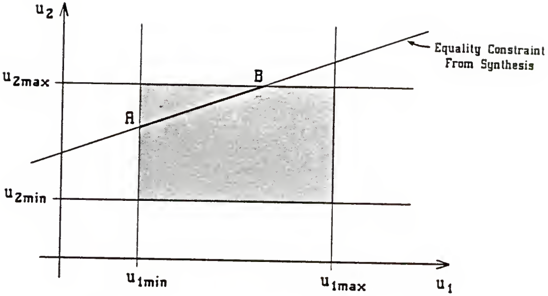
The inequality constraints (3.4.2) for each moving link of a mechanism experiencing optimization, together with the equality constraints obtained from dynamic synthesis (section 2.9) represent "necessary conditions" that must be satisfied. However, in some cases, these necessary conditions cannot be satisfied simultaneously by any of the design points, as illustrated in Fig. 3.2 for the design space of a generic objective function $F = F(u_1,u_2)$. Therefore, it is wise to check, right after synthesis, if there are any points in the design space that fulfill all the necessary conditions concurrently. How to accomplish this can probably be better explained by means of a specific illustration. Thus, taking as an example the shaking moment synthesis of the four-bar in section 2.11, the following constraints are obtained:

$$f^4_{312} = y^o_{12}$$

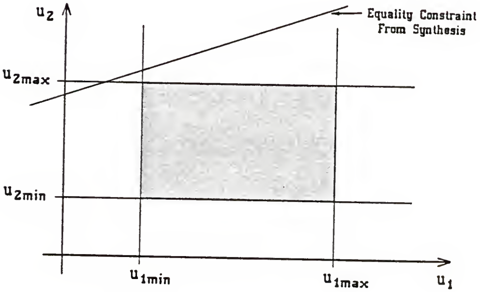
$$f^4_{412} = x^o_{12} + m_{23} - 2x^o_{23} + k^o_{23}$$

$$f^4_{223} = x^o_{23} - k^o_{23}$$

$$f^4_{323} = y^o_{23}$$



a) Only points on segment AB satisfy the inequality and equality constraints simultaneously.



b) There is no point in the design space satisfying the equality and inequality constraints simultaneously.

Figure 3.2 Two Examples of Possible Necessary Conditions for a Generic Objective Function $F=F(u_1,u_2)$

$$f^4_{443} = k^o_{23}+k^o_{43}$$

$$f^4_{343} = y^o_{43}$$

$$f^4_{243} = x^o_{43}-k^o_{43}$$

It should be noticed that all the f's at the left-hand-side of the equations above are known numbers. These equations and the inequality constraints for the three moving links can be satisfied simultaneously only if the following inequalities are true:

$$y^o_{12min} \leq f^4_{312} \leq y^o_{12max}$$

$$x^o_{12min}+m_{23min}-2x^o_{23max}+k^o_{23min} \leq f^4_{412} \leq$$

$$\leq x^o_{12max}+m_{23max}-2x^o_{23min}+k^o_{23max}$$

$$x^o_{23min}-k^o_{23max} \leq f^4_{223} \leq x^o_{23max}-k^o_{23min}$$

$$y^o_{23min} \leq f^4_{323} \leq y^o_{23max}$$

$$k^o_{23min}+k^o_{43min} \leq f^4_{443} \leq k^o_{23max}+k^o_{43max}$$

$$y^o_{43min} \leq f^4_{343} \leq y^o_{43max}$$

$$x^o_{43min}-k^o_{43max} \leq f^4_{243} \leq x^o_{43max}-k^o_{43min}$$

Essentially, the same checking procedure applies to other linkages and/or dynamic synthesis cases.

3.5 The Desirable Conditions

When designing a mechanism, the designer always "desires" to optimize certain important characteristics of the mechanism. To accomplish this, he must have some criteria by which the quality of such attributes can be measured. These criteria, which will be referred to here as "desirable conditions," are generally in conflict. Hence, the designer must look for a solution that reflects the best compromise for his dilemma. This can be achieved by means of an objective function composed of weighted desirable conditions, as described in the next section.

There are various well-known desirable conditions that can be used by a designer, but ideally, he should have the freedom to customize his own. In any case, a desirable condition should always fall within the following broad classification:

Class I - conditions dependent only on the geometry of the mechanism.

- Subclass Ia - conditions of Class I with no dynamic implications, e.g., the location of a fixed pivot.
- Subclass Ib - conditions of Class I with dynamic implications, e.g., the max. value of the g-function of an output link.

Class II - conditions dependent on the mass parameters of the mechanism, e.g., maximum value of the shaking force.

A complete package for the computer aided design (CAD) of mechanisms must include a module for kinematic optimization, followed by another for dynamic optimization. The former should have its objective function based on conditions from Class I, and the latter on conditions from Class II. As indicated in the rectangle at the bottom of Fig. 3.1, there must be a final comparative stage at the very end of the whole optimization process. In that stage, only conditions from Subclass Ia and Class II need to be considered.

It is opportune to emphasize that the conditions that belong to Subclass Ib represent dynamic "predictors," that is, they forecast the dynamics of a mechanism when only the geometry is known. Such predictors have been successfully applied to cam systems by Tesar and Matthew [51], and to Geneva mechanisms by Taat and Tesar [50]. Although they can act as powerful desirable conditions in a previous kinematic linkage optimization package, like the one described in [93], they do not possess the same value for the dynamic optimization package considered in this work. Here, it is preferable to address the dynamic properties of a linkage directly by means of the following desirable conditions from Class II:

a) Maximum magnitude of total[1] shaking force; ${}^{S}F^{t}_{max}$

b) Max. magnitude of total bearing reactions; F^{t}_{1max}, F^{t}_{2max}, etc.

c) Peak-to-peak value of total shaking moment; ${}^{S}M^{t}_{ptp}$

d) Root-mean-square value of total shaking moment; ${}^{S}M^{t}_{rms}$

e) Maximum absolute value of total shaking moment; ${}^{S}M^{t}_{max}$

f) Peak-to-peak value of total input torque; ${}^{I}T^{t}_{ptp}$

g) Root-mean-square value of total input torque; ${}^{I}T^{t}_{rms}$

h) Maximum absolute value of total input torque; ${}^{I}T^{t}_{max}$

The objective function in the software package to be described in the next chapter must be established from the desirable conditions above.

3.6 The Objective Function

Given a set of k conflicting desirable conditions (from the list presented in the previous section), it

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1 The adjective "total" in this list denotes summation of the contributions from internal masses and external loads.

becomes necessary to create the following objective function

$$S = \Sigma\ \mu_i S_i \ ; \ i = 1, 2, 3, \ldots, k \qquad (3.6.1)$$

where

S = cumulative score

i = desirable condition counter

S_i = score signifying performance of the linkage relative to the i^{th} condition

μ_i = weighting factor signifying importance of i^{th} condition relative to the other i-1 desirable conditions.

The k weighting factors are defined by the designer according to his experience and intuition.

The score S_i is calculated as follows:

$$S_i = \left(\frac{W_i - A_i}{W_i - B_i} \right)^{n_i} \quad ; \ 0 \leq S_i \leq 1 \qquad (3.6.2)$$

where

W_i = worst value for the i^{th} condition

B_i = best value for the i^{th} condition

A_i = actual value for the i^{th} condition ($B_i \leq A_i \leq W_i$)

n_i = exponent allowing scores to be assigned in a non-linear manner ($n_i > 0$)

Since, in this work, $B_i = 0$ always, Eq. (3.6.2) becomes

$$S_i = \left(1 - \frac{A_i}{W_i}\right)^{n_i} \quad ; \; 0 \leq A_i \leq W_i \qquad (3.6.3)$$

According to Eq. (3.6.3) and its graphical representation in Fig. 3.3., the lower the actual value A_i, the more the score S_i approaches 1, its highest possible value. The exponent n_i must also be set by the designer in conformity with his experience and intuition.

It is desirable to normalize the cumulative score S, so that its highest possible value is always the same, under the unrealistic but appropriate assumption that all the S_i's can simultaneously attain the value 1. In this work, such referential maximum value has been chosen to be 100. Thus, from Eq. (3.6.1), it is possible to write

$$\mu_1 + \mu_2 + \ldots + \mu_k = 100 \qquad (3.6.4)$$

If $\mu'_1, \mu'_2, \ldots, \mu'_k$ are the values chosen by the designer for the weighting factors of all the k desirable conditions, such that

$$0 \leq \mu'_i \leq 10 \quad ; \; i = 1, 2, \ldots, k$$

then

$$\mu_i = (\mu'_i/\mu'_1)\mu_1 \quad \text{if } \mu'_1 \neq 0 \; ; \; i = 2, 3, \ldots, k \qquad (3.6.5)$$

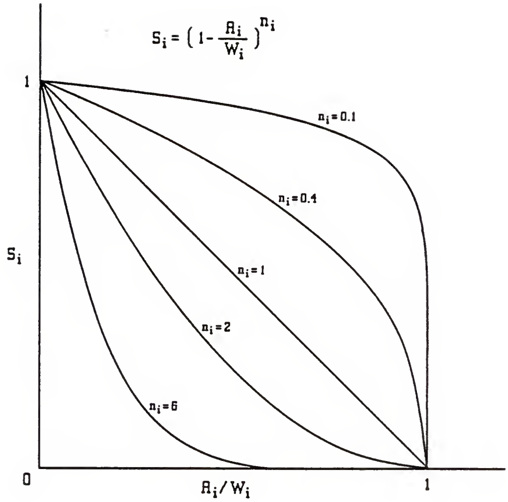


Figure 3.3 Some Possible Scoring Curves for Desirable Condition i

Substituting Eqs. (3.6.5) into Eq. (3.6.4) yields

$$\mu_1(\mu_1' + \mu_2' + \ldots + \mu_k') \div \mu_1' = 100$$

or

$$\mu_1 = 100\mu_1' \div \Sigma\mu_i' \quad ; \; i = 1 \text{ to } k \tag{3.6.6}$$

Extending this result to all the other weighting factors, gives

$$\mu_j = 100\mu_j' \div \Sigma\mu_i' \quad ; \; i = 1 \text{ to } k \tag{3.6.7}$$

$$j = 1, 2, \ldots, k$$

CHAPTER IV
AN INTERACTIVE CAD PACKAGE

The theory presented in Chapters II and III has been implemented in an interactive software package, which is currently applicable only to four-bar mechanisms. However, it can be expanded to encompass other linkages, with relative ease. This chapter is not intended to be a user's manual. In fact, the package is intuitive and self-explanatory, but it is recommended that the user be aware of the flowchart presented in the preceding chapter.

4.1 General Description of the CAD Package

The software package addressed by this chapter has been written in APL which is a computer language very suitable for manipulation of arrays. The APL functions that compose the package can be classified into two groups, namely, the "Graphics System" and the "Application Program." The Graphics System consists of the primitive graphic functions used to create graphics elements such as segments of lines, polygons, circles, and character strings on the screen of a display terminal. These functions, which are listed in Appendix E, are very much dependent on the low-level architecture of the terminal

and the X-Y coordinate system of the physical screen. They are compatible with the Tektronix 4010/4012/4013/4014 and 4015 graphics terminals (Fig. 4.1), or any other terminal capable of simulating a Tektronix from the 4010-series mentioned above. The Application Program consists of the functions which directly implement the theory discussed in Chapters II and III. These functions are listed in Appendix F.

4.2 The Application Program

The Application Program is based on the flowchart in Fig. 3.1 which has been reproduced for convenience in this chapter as Fig. 4.2. This chart is not complete in details. It has been created to illustrate the main logical decision making stages involved in the package.

Figure 4.3 is a photograph of part of the screen of a Tektronix 4015 terminal. It shows the initial stage of the computer aided design process where information about the geometry and motion of the four-bar is entered into the computer. This photograph does not show the image of a referential four-bar linkage drawn at the lower right corner of the screen. Such image was similar to the one shown in Fig. 4.4. An important feature of the Application Program is that it allows the user to choose the locations for the x-y coordinate systems of the moving links. The user may, for example, choose the local

Figure 4.1 Tektronix 4015 Computer Display Terminal

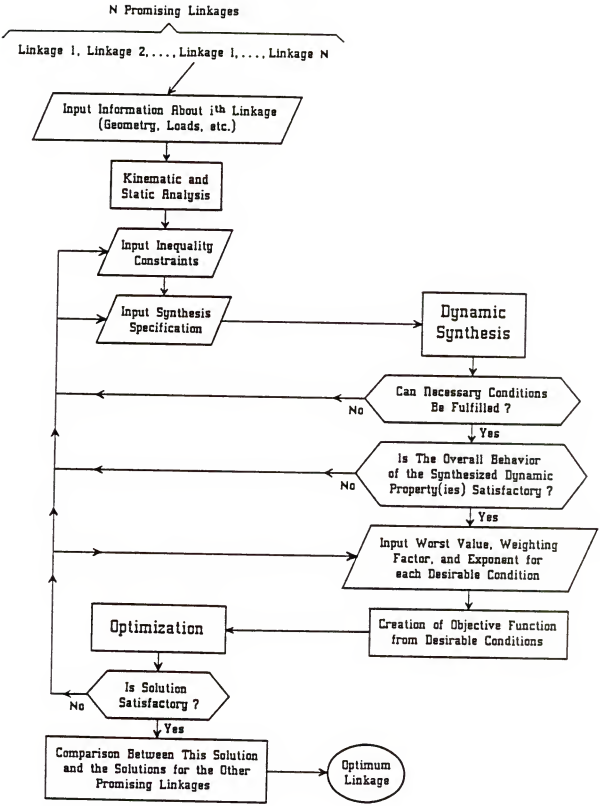


Figure 4.2 Integration of Dynamic Synthesis and Optimization in the CAD Package

```
ENTER COORDINATES OF PIVOTS 1 AND 4 (U1 V1 U4 V4):
□:
      0 0 5 0
FNTER LENGTHS OF CRANK, COUPLER, AND LINK 43
(L12 L23 L43):
□:
      2 4 4
WHAT IS THE TYPE OF DYAD 234?
  1) +1
  2) -1
  3) HELP
3
THE TYPE IS +1 IF WHEN YOU STAND AT JOINT 2 AND
LOOK AT JOINT 4, JOINT 3 IS AT YOUR LEFT.
OTHERWISE THE TYPE IS -1 .

WHAT IS THE TYPE OF DYAD 234?
  1) +1
  2) -1

1
CHOOSE UNIT FOR ANGULAR VELOCITY
  1) RADIANS PER SEC.
  2) RPM
1
NOW ENTER INPUT ANGULAR VELOCITY
(+) IF CCW ; (-) IF CW:
□:
      1
WHAT IS THE INPUT ANGLE INCREMENT IN DEGREE ?
(A SUBMULTIPLE OF 360 IS RECOMMENDED)
□:
      1■
```

Figure 4.3 Initial Stage of the CAD Package

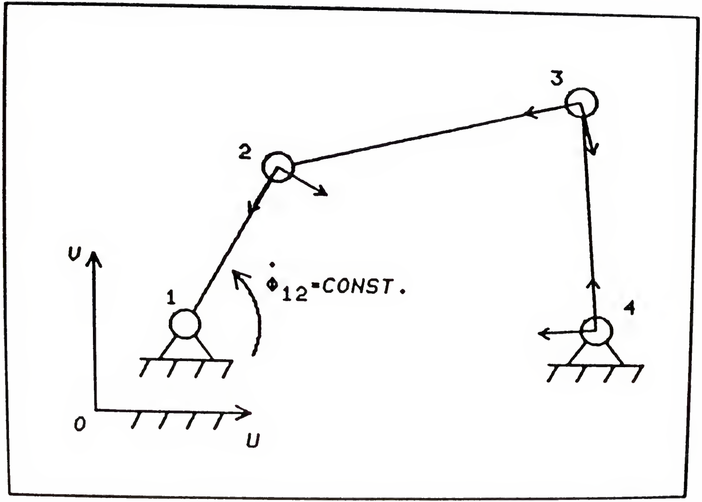


Figure 4.4 A Four-Bar Linkage and a Possible Set of Local Coordinate Systems

systems shown in Fig. 4.4. For all subsequent input and output operations, the Application Program performs all the necessary transformations between the user's coordinate systems and the set of x-y coordinate systems which is internal to the program and completely transparent to the user.

The rest of this chapter will address some peculiarities of the application program with respect to external loads, synthesis, optimization, and the basic mass parameters.

4.3 External Loads

The Application Program offers the user two ways for entering information about external loads. In one of them, the numerical data for the external loads must be typed in one at a time for each position of the input crank. However, if the data to be entered assume a constant value for certain interval of motion, this value needs to be entered only once. The other way to input the external loads into the computer is by means of a "calculator" portrayed on the screen, and the crosshair cursor, as shown in Fig. 4.5. The cursor can be positioned by two thumbwheels at the far right of the keyboard (Fig. 4.1), or optional joystick. This calculator uses the RPN logic and has a stack with four storage registers (X, Y, Z and T). Every time a number is

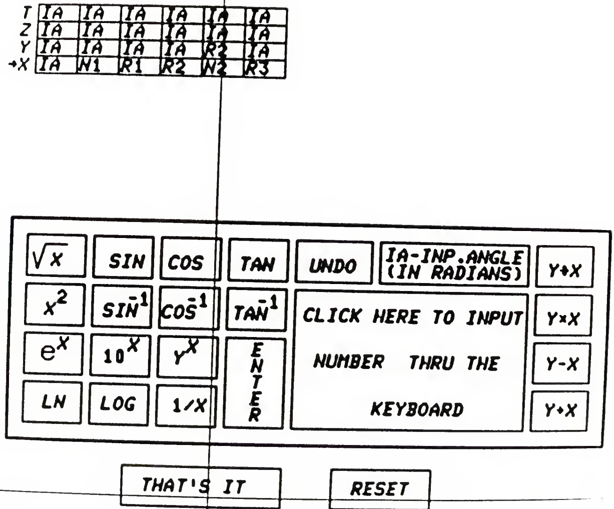


Figure 4.5 A Computer Generated Calculator

entered into the X register, or an operation is performed, a new representation of the stack appears near the previous ones. Figure 4.5 shows six such representations, where IA, N1, and R1 signify input angle, first number entered through the keyboard of the terminal, and the result of first operation performed by the calculator, respectively. The calculator is useful when there is a known function of the input angle from which the data to be entered can be calculated for a specified interval of motion. For example, Fig. 4.6 shows the plot of a fictitious load versus input angle created by means of the calculator. It is important to understand that the computer does not retain the analytical expressions for each of the "function segments," but instead it keeps only the function value for each of the equally spaced positions of the input crank.

Either the global coordinate system or the appropriate local coordinate system can be used as a reference for an external force. In addition, the point of application of a force on a moving link can be continuously changing.

When the static analysis is finished, the application program asks the user if he wants to see the plot of the shaking force, shaking moment, input torque or any of the bearing reactions, due to the external loads.

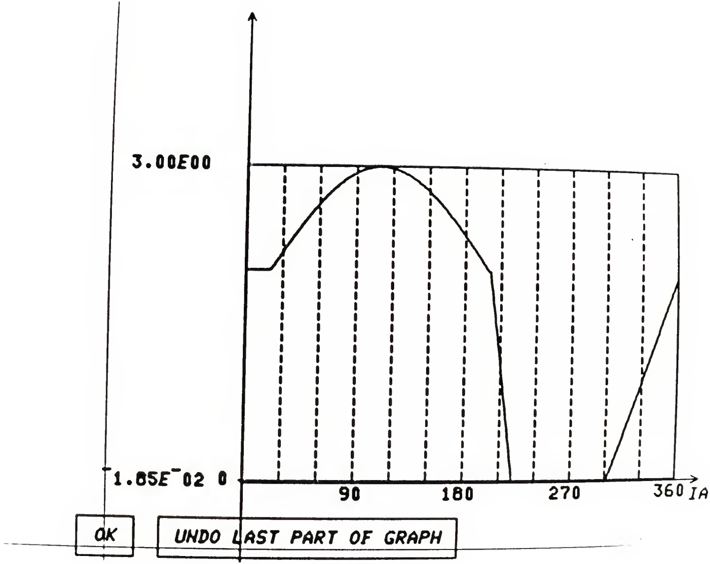


Figure 4.6 A Fictitious Load Entered by Means of the Calculator

4.4 Dynamic Synthesis

Even though it is not explicitly indicated by the flowchart in Fig. 4.2, there is a stage in the Application Program, before dynamic synthesis, where the user is allowed to enter mass parameters for the four bar, in case he wishes to do so. This is useful if the user wants to balance an existent linkage or if he has a good feeling about what the optimum mass parameter might be.

As demonstrated in Chapter II, either shaking moment synthesis or the double synthesis of shaking force and input torque is possible for a four-bar linkage. When the synthesis stage is reached, the Application Program displays the curve of the dynamic property to be synthesized that results from the external loads and/or the internal masses of the four-bar.

Figure 4.7 shows a shaking moment plot and seven precision points selected with the crosshair cursor, and Fig. 4.8 shows the synthesized shaking moment curve passing through all the seven precision points. Synthesis can be performed as many times as the user wants until a satisfactory curve is obtained. If more than seven precision points are selected for shaking moment synthesis, and more than three for input torque synthesis, they are approximated in the least squares sense. Figure 4.9 shows a shaking force hodograph and two precision points, and Fig. 4.10 shows the synthesized shaking force.

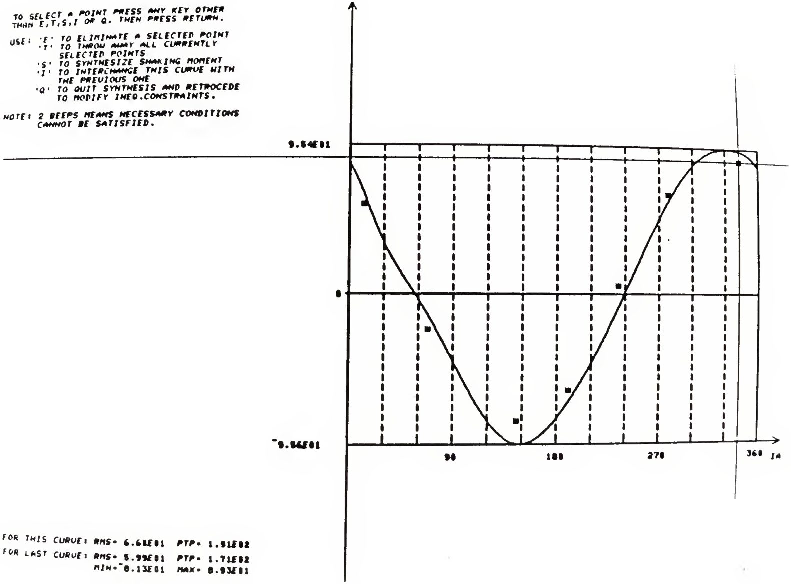
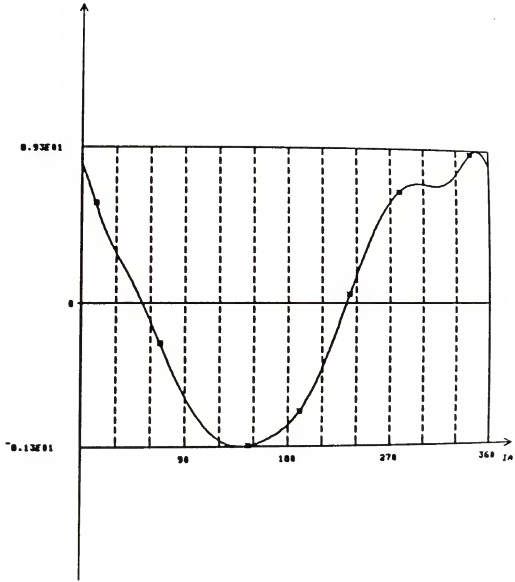


Figure 4.7 A Shaking Moment Plot and Seven Precision Points

DOES THIS CURVE SATISFY YOU?



FOR THIS CURVE: RMS= 5.99E01 PTP= 1.71E02
FOR LAST CURVE: RMS= 6.60E01 PTP= 1.91E02
MIN=-9.56E01 MAX= 9.54E01

Figure 4.8 A Synthesized Shaking Moment

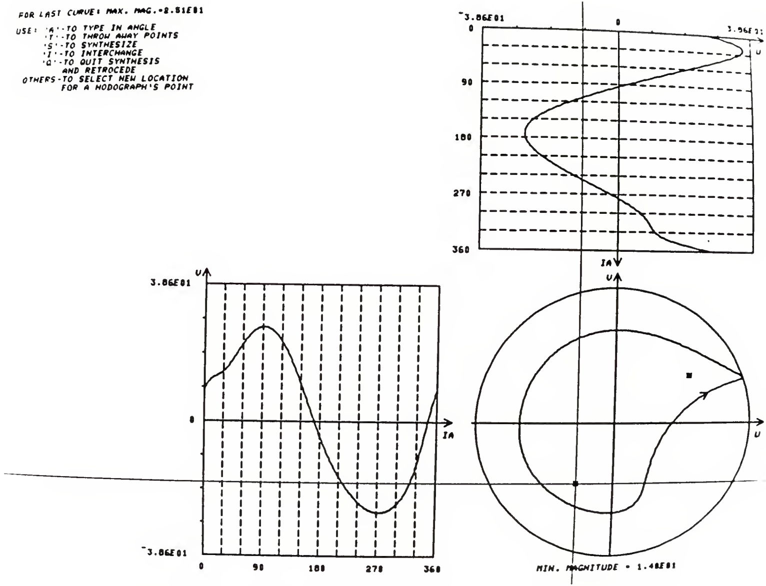


Figure 4.9 A Shaking Force Hodograph and Two Precision Points

FOR LAST CURVE: MAX. MAG.=3.86E01
DOES THIS CURVE SATISFY YOU?

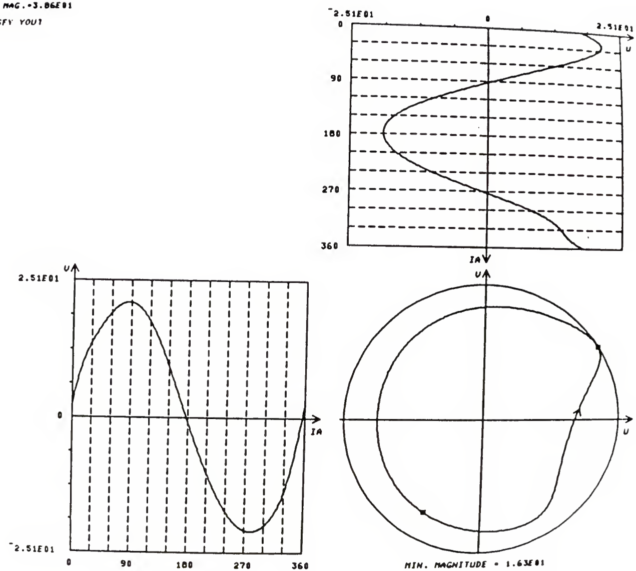


Figure 4.10 A Synthesized Shaking Force

4.5 Optimization

Before the optimization stage can be initiated, the user has to define the objective function. Therefore, for each of the desirable conditions considered (from the list in section 3.5), the user must enter the worst value, weighting factor, and the exponent of Eq. (3.6.3). To select this exponent, the user indicates a trial value, and the Application Program plots the corresponding score curve on the screen (Fig. 4.11). This process can be repeated as many times as necessary until a value that meets the user's intuition is found.

As mentioned in the last chapter, optimization in the package is performed by means of a random search method improved by the techniques of "sequential filtering" and "grid expansion." Thus, for every design point (m_{23}, x^o_{23}) belonging to a specified grid in the region of the design space defined by the constraints

$$m_{23min} \leq m_{23} \leq m_{23max} \qquad (4.5.1a)$$

and

$$x^o_{23min} \leq x^o_{23} \leq x^o_{23max} , \qquad (4.5.1b)$$

the equality constraints obtained from synthesis are solved sequentially, and each calculated mass parameter is in turn compared to its specified limits. If the design point is not rejected during this process, the Application Program proceeds to calculate the desirable conditions

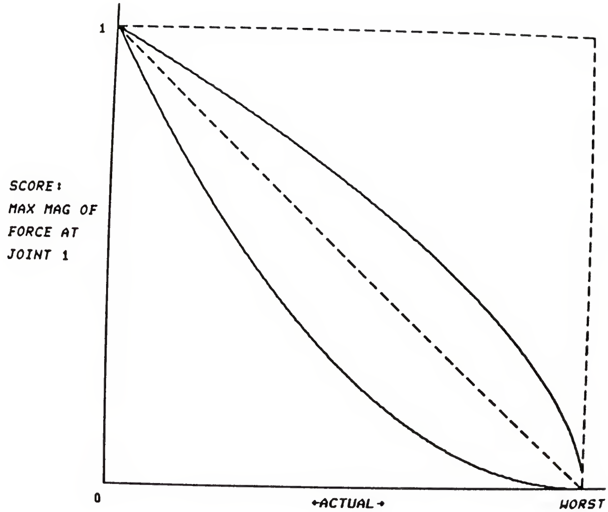


Figure 4.11 Two Score Curves (n_i = 0.6 and 2)

also sequentially. However, before the calculation of condition i+1, the condition i is checked against its permissible worst value. If the constraint imposed by this extreme value is not satisfied, the design point is rejected and a new one selected. The "grid expansion" technique (section 3.1) allows the user to specify a two-dimensional grid in the region of the design space defined by the inequalities (4.5.1), as indicated in Fig. 4.12. A second and more refined grid can be generated about the best point from the first grid. This process can be repeated as many times as desirable. At the end of the optimization stage, the optimum mass parameters found are displayed on the screen, as shown in Fig. 4.13.

4.6 The Transformation from Lumped to Basic Mass Parameters

The lumped mass parameters defined in section 2.10 greatly simplify the formulation and solution of the synthesis and optimization problems. In fact, for mechanisms more complex than the four-bar, these problems would become extremely complicated without the introduction of such parameters. However, since the basic mass parameters are the ones that ultimately interest the designer, the inverse transformation from the "lumped space" to the "basic space" is necessary. In the package, this transformation is performed during the optimization phase for every point of the lumped design space that

```
THE FREE MASS PARAMETERS FOR THE OPTIMIZATION
PROCESS THAT IS ABOUT TO BEGIN ARE:'M23' AND 'XO23'
WHERE,
        2≤M23≤30 AND ¯15≤XO23≤45

PLEASE ENTER A PAIR OF NUMBERS INDICATING
HOW MANY EQUALLY SPACED VALUES IN THE ABOVE
INTERVALS YOU WANT TO CONSIDER, INITIALLY, FOR
'M23' AND 'XO23', IN THAT ORDER:
⎕:
      20 20■
```

Figure 4.12 The Specification of a Grid in the Design Space

```
THE OPTIMUM MASS PARAMETERS ARE:

 7.584549 ≤M12≤ 20
 X012=10.531917
 Y012= .09360476

 M23=3.4736842
 X23=2.3554789
 Y23=.21164587
 K23=2.2828521

 3.2293509 ≤M43≤ 30
 X043=1.5690468
 Y043=.62457996
 K043=3.3556239

NOTE: K12 CAN ASSUME
      ANY ARBITRARY VALUE.

PLEASE, CHOOSE ONE ALTERNATIVE:
  1) RETROCEDE TO MODIFY MASS PARAMETERS' LIMITS
  2) RETROCEDE TO REDO SYNTHESIS
  3) RETROCEDE TO MODIFY OBJECTIVE FUNCTION
  4) END
■
```

Figure 4.13 The End of the Optimization Stage

satisfies the necessary conditions imposed on the lumped mass parameters. The reason why the transformation is not performed at the end of optimization, for the optimum solution only, is because the transformed point may not satisfy all the side constraints on the basic mass parameters, as exemplified in Fig. 4.14. The following algorithm has been included in the Application Program to perform the transformation above:

Link 23 (the coupler):[1]

i) Calculate $\bar{x}_{23}$, $\bar{y}_{23}$, and $\bar{k}_{23}$ from known m_{23}, x^o_{23}, y^o_{23}, and k^o_{23}.

ii) Proceed if side constraints for link 23 are satisfied, otherwise reject point.

Link 12 (the input link):

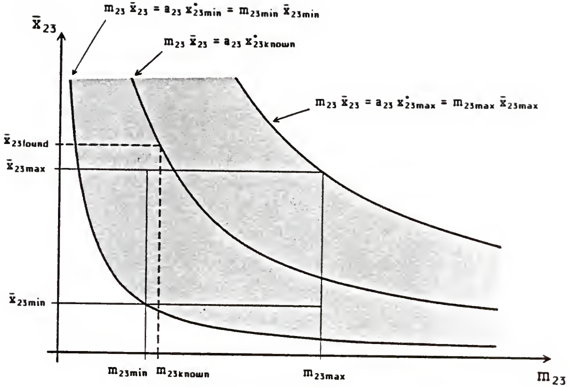
i) Find the interval Δm_{12x} from known x^o_{12}, as illustrated in Fig. 4.15.

Note: $\Delta m_{12x} = [m_{12min}, m_{12max}]$ if $x^o_{12} = 0$.

ii) Similarly, find Δm_{12y} from known y^o_{12}.

---

1 The reference for this algorithm is the four-bar drawn on page 47.



NOTE: $\dot{x}_{23known}$ satisfies the side constraint

$$\dot{x}_{23min} \le \dot{x}_{23} \le \dot{x}_{23max}$$

But $\bar{x}_{23found}$ does not satisfy the constraint

$$\bar{x}_{23min} \le \bar{x}_{23} \le \bar{x}_{23max}$$

Figure 4.14 An Illustrative Example of the Calculation of $\bar{x}_{23}$ from known m_{23} and x^{o}_{23}

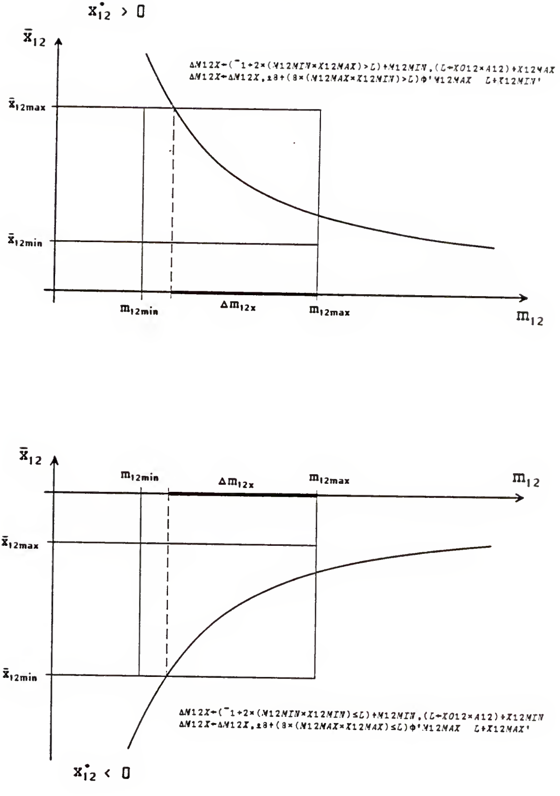


Figure 4.15 Two Examples of the Calculation of Δm_{12x}

iii) Find $\Delta m_{12} = \Delta m_{12x} \cap \Delta_{12y}$.
If $\Delta m_{12} = \phi$ (empty set), reject point.

Link 43:

i) Find Δm_{43x} from known x^{o}_{43} by the same method used for link 12.

ii) Similarly, find Δm_{43y} from known y^{o}_{43}.

iii) Find $\Delta m'_{43} = \Delta m_{43x} \cap \Delta m_{43y}$.
If $\Delta m'_{43} = \phi$, reject point.

iv) Find Δm_{43k} from known x^{o}_{43}, y^{o}_{43}, and k^{o}_{43}, con-considering the following

$$k^{o}_{43} = m_{43}(\bar{x}^{2}_{43}+\bar{y}^{2}_{43}+\bar{k}^{2}_{43}) \div a^{2}_{43} \qquad (4.6.1)$$

Substituting $\bar{x}^{2}_{43} = [(a_{43}x^{o}_{43}) \div m_{43}]^{2}$ and

$\bar{y}^{2}_{43} = [(a_{43}y^{o}_{43}) \div m_{43}]^{2}$ into Eq. (4.6.1), gives

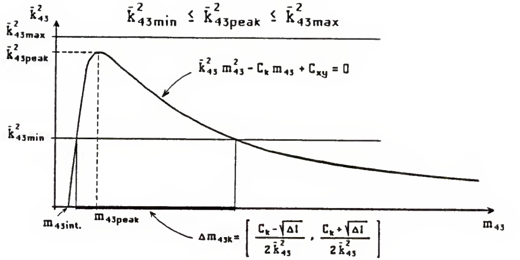
$$a^{2}_{43}k^{o}_{43} = a^{2}_{43}[(x^{o}_{43})^{2}+(y^{o}_{43})^{2}]m^{-1}_{43}+m_{43}\bar{k}^{2}_{43} \qquad (4.6.2)$$

$$\therefore \quad C_{k} = C_{xy}m^{-1}_{43}+m_{43}\bar{k}^{2}_{43} \qquad (4.6.3)$$

where

$$C_{k} = a^{2}_{43}k^{o}_{43} > 0 \quad \text{and} \quad C_{xy} = a^{2}_{43}[(x^{o}_{43})^{2}+(y^{o}_{43})^{2}] \geq 0$$

If $C_{xy} = 0$, find Δm_{43k} by the same method used to find Δm_{43x} and Δm_{43y}. Otherwise refer to Fig. 4.16.



$$m_{43int.} = \frac{C_{xy}}{C_k} \ge 0$$

$$m_{43peak} = 2\,\frac{C_{xy}}{C_k} \quad \text{and} \quad \bar{k}^2_{43peak} = \frac{1}{4}\,\frac{C_k^2}{C_{xy}}$$

$$\Delta 1 = C_k^2 - 4\bar{k}^2_{43min}\,C_{xy}$$

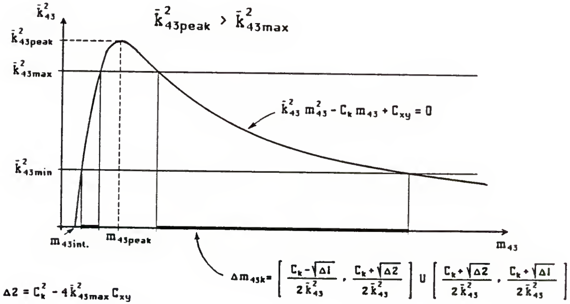


Figure 4.16 The Calculation of Δm_{43k}

Note: If $\bar{k}^2_{43peak} < \bar{k}^2_{43min}$, reject point (Fig. 4.15).

v) Find $\Delta m_{43} = \Delta m'_{43} \cap \Delta m_{43k}$.
If $\Delta m_{43} = \phi$, reject point; otherwise it can be accepted.

The algorithm just presented can be applied to any linkage since each moving link of a given mechanism will always correspond to one of the three types of links considered above.

CHAPTER V
EXAMPLES

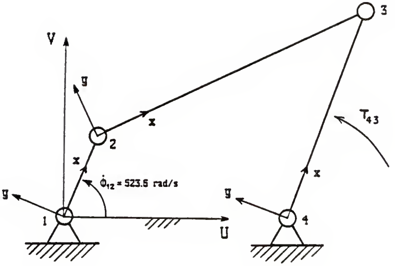
In this chapter, the redistribution of the internal masses of a fully force balanced four-bar linkage acted by an external torque will be considered to illustrate the two basic ways in which the CAD package described in Chapter IV can be applied. This will be performed by means of one example involving the double syntheses of the input torque and shaking force of the fully force balanced four-bar, and another example addressing the shaking moment synthesis of the same mechanism.

5.1 <u>The Known Linkage and Load</u>

The mass parameters of the mechanism in Fig. 5.1 make the first four functions given on page 45 vanish, that is,

$$\mathrm{Re}(f^2_{412}) = \mathrm{Im}(f^2_{412}) = \mathrm{Re}\,(f^2_{443}) = \mathrm{Im}(f^2_{443}) = 0$$

Thus, the inertial shaking force acting on the foundation of the mechanism is zero, and so is the shaking force due to the external torque T_{43}. This torque has the following specification:



Parameter	Link			
	12	23	43	14
length [mm]	25	102	76	76
mass [Kg]	0.150	0.103	0.300	—
$\bar{x}$ [mm]	-8.9	51	-13	—
$\bar{y}$ [mm]	0	0	0	—
$\bar{k}$ [mm]	20	51	36	—

Note : The shaking moment reference point for this chapter is $(U,V) = (0.038\ m\ ,0)$

Figure 5.1 A Fully Force Balanced Four-Bar Linkage

$T_{43} = 0$; $\phi_{12} = 0^{\circ}$ to 35°

$T_{43} = -70\cos[(\phi_{12}-0.6109) \div 0.5694]+70$ Nm; $\phi_{12} = 35^{\circ}$ to 240°

$T_{43} = 0$; $\phi_{12} = 240^{\circ}$ to 360°

where ϕ_{12}, in the argument of the cosine function above, must be in radians. The graphical representation of T_{43} is presented in Fig. 5.2.

Figures 5.3 and 5.4 show the total shaking moment and the total input torque of the mechanism described in Fig. 5.1. The maximum values of the bearing reactions are

$$F_{1max} = F_{4max} = 5390 \text{ N}$$

$$F_{2max} = 5720 \text{ N}$$

$$F_{3max} = 4870 \text{ N}$$

5.2 Example 1: Input Torque and Shaking Force Synthesis

The following assumptions apply to this example:

1. The foundation of the mechanism is massive, and the shaking force is not a very important property.
2. It is desirable to reduce the root-mean-square value, the peak-to-peak value, and the maximum absolute value of the input torque.

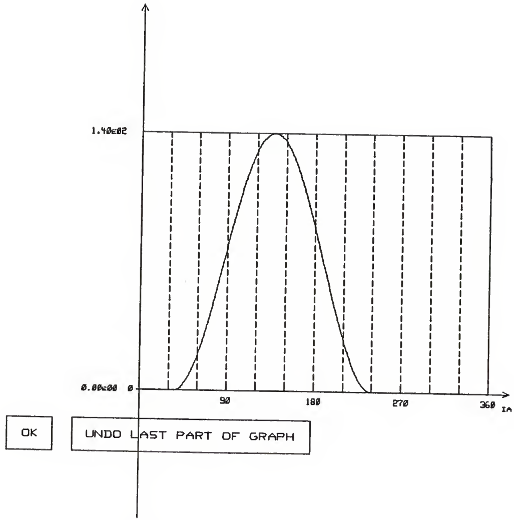


Figure 5.2 External Torque T_{43} in Nm

PRESS 'RETURN' TO CONTINUE

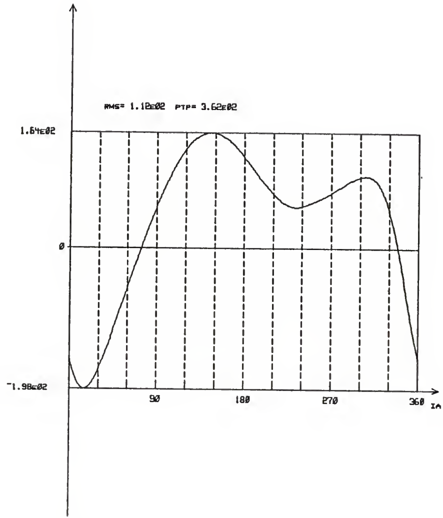


Figure 5.3 Total Shaking Moment in Nm of the Fully Force Balanced Four-Bar

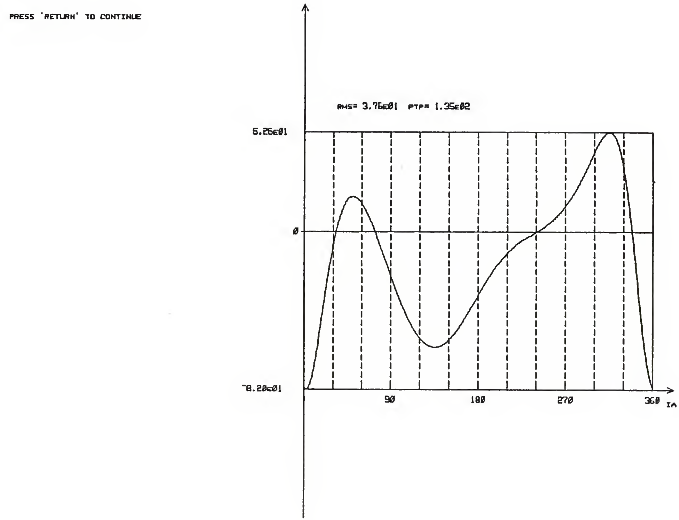


Figure 5.4 Total Input Torque (in Nm) of the Fully Force Balanced Four-Bar

Figures 5.5 and 5.6 show the input torque and shaking force synthesized by means of the CAD package described in the last chapter. In accordance with Chapter II, the number of free design variables was reduced from 9 to 2, by the synthesis process. The mass parameters that were carried over to the optimization phase were m_{23} and x^{o}_{23}. The objective function given by Eqs. (3.6.1) and (3.6.3) was specified as indicated in Table 5.1.

Table 5.1
Specification of Objective Function
for Example 1

Desirable Condition*	μ'_i	W_i	n_i
F^t_{jmax} ; j=1,...,4	10	5000 N	1
${}^{s}M^t_{ptp}$	6	350 Nm	1
${}^{s}M^t_{rms}$	6	100 Nm	1
${}^{s}M^t_{max}$	6	190 Nm	1

* Section 3.5.

The optimum solution obtained is presented in Table 5.2. The corresponding shaking moment plot is shown in Fig. 5.7, and the maximum values of the bearing reactions are

$$F_{1max} = 2990 \text{ N}$$
$$F_{2max} = 2840 \text{ N}$$

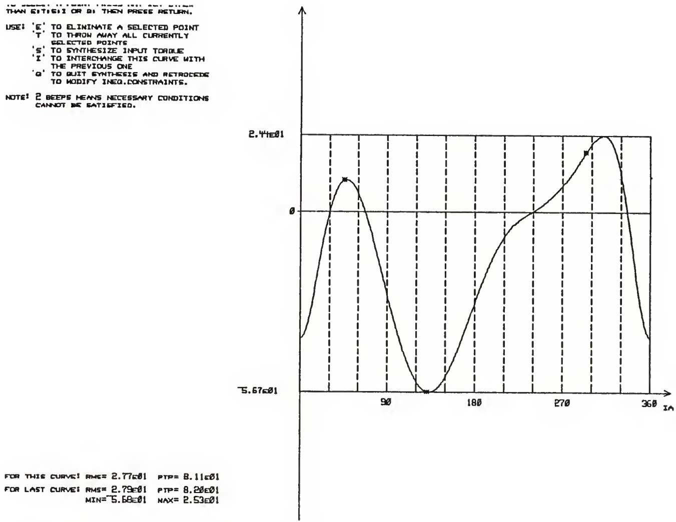


Figure 5.5 Total Synthesized Input Torque (in Nm) Obtained in Example 1

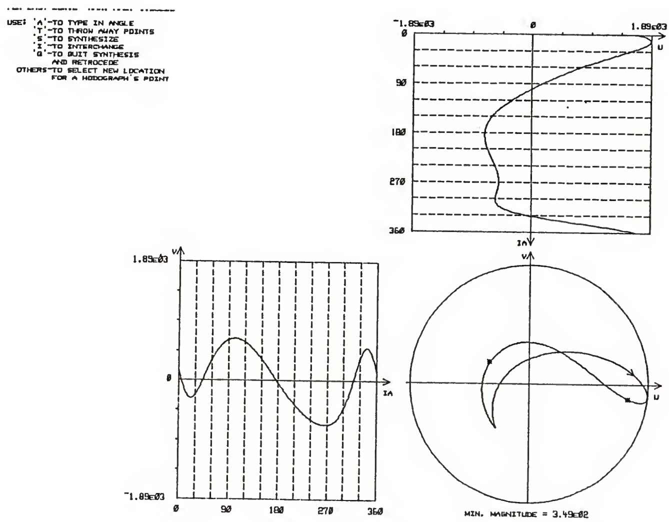


Figure 5.6 Synthesized Shaking Force (in N) Obtained in Example 1

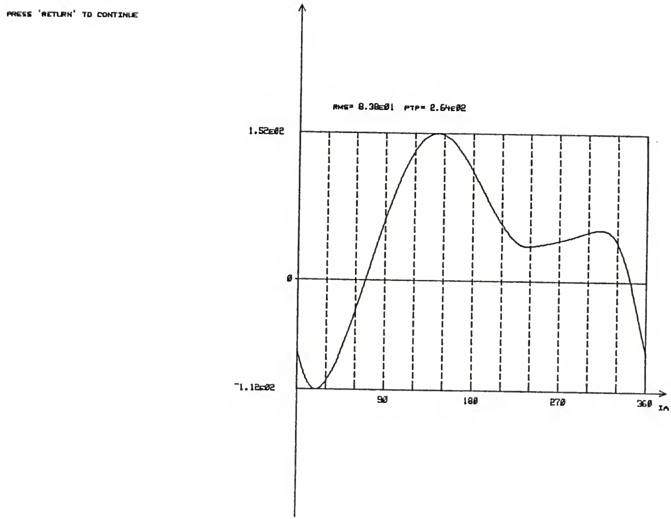


Figure 5.7 Total Shaking Moment (in Nm) for the Optimum Solution in Example 1

$$F_{3max} = F_{4max} = 2210 \text{ N}$$

Table 5.2
Optimum Solution for Example 1
(Score = 40)

Parameter	Link 12	23	43
mass [Kg]	0.045	0.0787	0.066
$\bar{x}$ [mm]	12.7	50.8	38.1
$\bar{y}$ [mm]	2.1	0.5	1.6
$\bar{k}$ [mm]	--	34.9	27.9

5.3 Example 2: Shaking Moment Synthesis

If the foundation of the four-bar in Fig. 5.1 is not massive, the shaking moment (Fig. 5.3) may cause too much vibration on the surroundings of the mechanism. The synthesized shaking moment shown in Fig. 5.8 represents an attempt to reduce these undesirable vibrations. As in example 1, the number of free design variables was also reduced to 2 by this synthesis process. The objective function for the optimization stage was specified as indicated in Table 5.3.

The optimum solution obtained is presented in Table 5.4. The corresponding input torque and shaking force plots are shown in Figs. 5.9 and 5.10, and the maximum values of the bearing reactions are

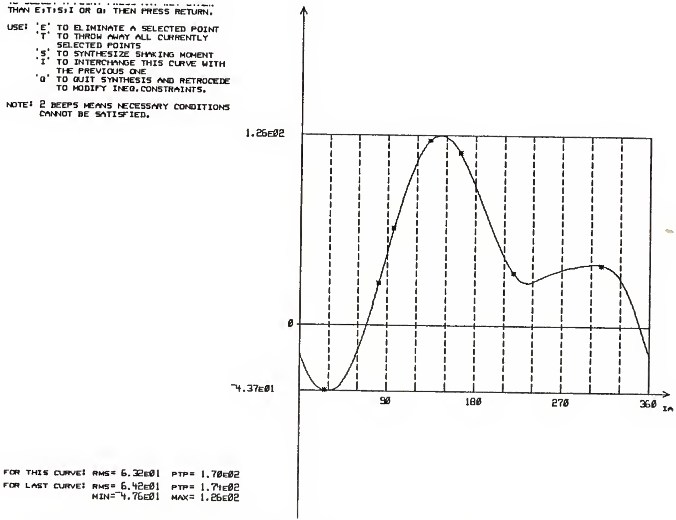


Figure 5.8 Synthesized Shaking Moment (in Nm) Obtained in Example 2

Table 5.3
Specification of Objective Function
for Example 2

Desirable Condition*	μ_i'	W_i	n_i
F^t_{1max}, F^t_{4max}	6	5000 N	1
F^t_{2max}, F^t_{3max}	5	5000 N	1
${}^{s}F^t_{max}$	10	4000 N	2
${}^{I}T^t_{ptp}$	8	130 Nm	1
${}^{I}T^t_{rms}$	8	35 Nm	1
${}^{I}T^t_{max}$	8	80 Nm	1

* Section 3.5.

Table 5.4
Optimum Solution for Example 2
(Score = 49.3)

Parameter	Link 12	23	43
mass [Kg]	0.062	0.0647	0.051
$\bar{x}$ [mm]	11.2	1.0	17.2
$\bar{y}$ [mm]	3.3	2.1	10.9
$\bar{k}$ [mm]	--	36.2	32.7

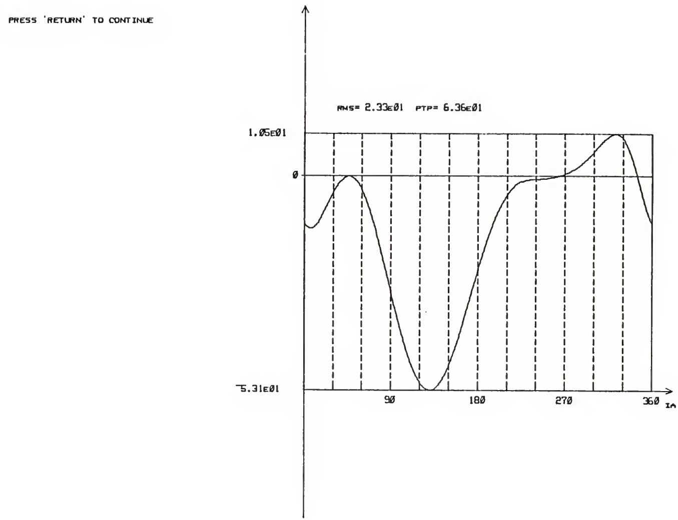


Figure 5.9 Total Input Torque (in Nm) for the Optimum Solution in Example 2

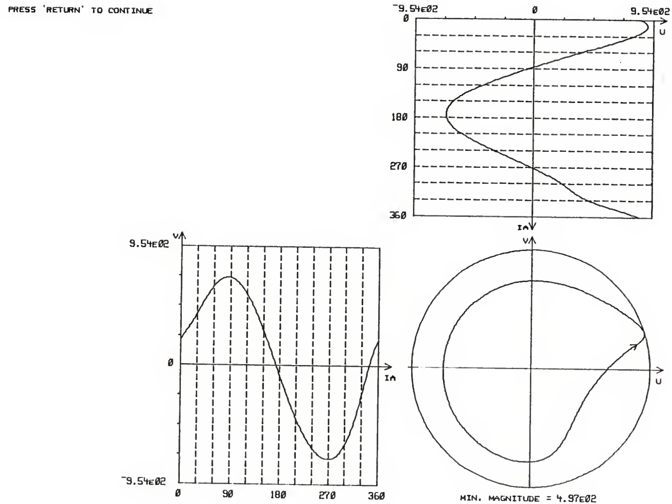


Figure 5.10 Shaking Force (in N) for the Optimum Solution in Example 2

$$F_{1max} = 2380 \text{ N}$$

$$F_{2max} = 2230 \text{ N}$$

$$F_{3max} = 2080 \text{ N}$$

$$F_{4max} = 2040 \text{ N}$$

5.4 Comments

The previous examples have been included in this work to illustrate the applicability of the CAD package described in the last chapter. Clearly, the solution obtained in example 2 is better than the one achieved in example 1. However, improved solutions could very probably be obtained, in both examples, if more time had been dedicated to the synthesis and optimization processes. This was not possible due to high instability of the digital computer used.

CHAPTER VI
CONCLUSIONS AND RECOMMENDATIONS

6.1 Conclusions

As indicated in the introductory chapter of this work, mechanisms and machines play a very important role in modern life. Despite the existence of innumerable sophisticated mechanical systems nowadays, there is still a need for powerful and efficient means to design faster and more precise machines. It is the belief of the author that this dissertation represents a substantial contribution to the fulfillment of such need. The theory contained in Chapter II is a profound rearrangement of the work done by Elliott in Ref. [1]. The concepts of point masses and composite point masses greatly facilitate the derivation of the expressions for the shaking force, shaking moment, and input torque of any planar linkage. Thus, by just looking at the diagram of a mechanism, one can immediately write down these expressions in a form free of linear dependencies and hence suitable for dynamic synthesis. The lumped mass parameters defined in section 2.10 considerably simplify the formulation and solution of the synthesis and optimization problems. The optimization technique presented in Chapter III allows the designer to

impose inequality constraints on the mass parameters so that the chance of obtention of an impractical solution is highly reduced. The CAD package described in Chapter IV confirmed the validity and usefulness of the theory contained in Chapters II and III.

6.2 Restrictions and Recommendations for Future Works

The following items are limitations of this work and recommendations for future works:

1. A future research to extend the method contained here to systems composed of several inter-connected planar linkages is highly desirable. An extension to encompass spatial linkages would also be valuable.
2. A sound combination of dynamic synthesis and spring synthesis would be very useful, specially for balancing mechanisms that operate under large loads that cannot be compensated by the internal masses only.
3. The CAD package presented here is applicable only to four-bar mechanisms. Future software packages should consider other linkages besides the four-bar.
4. The "calculator" implemented in the CAD package described here cannot be easily used to enter a load which is a function of, for example, an

output link angle. In addition, the package does not permit the user to enter an external load by means of a graphics tablet, if the plot of such load is available.

5. A profound investigation to improve the efficiency of the optimization phase is recommended.
6. The use of a rapid raster display terminal instead of a storage tube one is also recommended, since the former is generally more suitable for dynamic picture manipulation and selective erasure of just some parts of the picture on the screen.
7. An important part of the design of a mechanism consists of the definition of the complete shape of its moving links, once the optimum mass parameters are known, and the verification for strength of these links. A complete CAD system for mechanism design should offer powerful interaction capabilities during this crucial part of the design process. Additionally, the real-time animation of the solution mechanism would certainly be appreciated by the designer.
8. Finally, the designer should have some flexibility to customize his own desirable conditions to be used by the optimization procedure.

APPENDIX A
DERIVATION OF INERTIAL INPUT TORQUE BY MEANS OF KINEMATIC INFLUENCE COEFFICIENTS

The concept of velocity and acceleration influence coefficients can be used profitably for kinematic and dynamic analysis of multi-degree of freedom mechanisms [21,32,40,45]. Here, this concept will be used for the derivation of the equivalent inertia (I^*_{pq}) at the input shaft of one-degree of freedom linkages with constant input angular velocity. Once I^*_{pq} is obtained in a proper form, the expression for the inertial input torque can easily be deduced.

If the formulation derived in Fig. A.1 for the generic point "r" is applied to the center of gravity, the kinetic energy of link pq can be written as

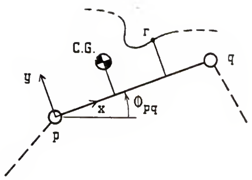
$$KE_{pq} = 0.5\{m_{pq}[(\bar{g}^u_{pq})^2+(\bar{g}^v_{pq})^2]+m_{pq}\bar{k}^2_{pq}(g^\phi_{pq})^2\}\omega_i^2 = 0.5\ I^*_{pq}\omega_i^2$$

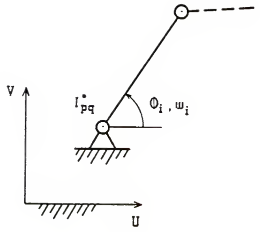
$$\therefore \qquad I^*_{pq} = m_{pq}[(\bar{g}^u_{pq})^2+(\bar{g}^v_{pq})^2]+m_{pq}\bar{k}^2_{pq}(g^\phi_{pq})^2 \qquad (A.1)$$

Substituting

$$\bar{g}^u_{pq} = g^u_p-g^\phi_{pq}(\bar{x}_{pq}s_{pq}+\bar{y}_{pq}c_{pq})$$

and





$$\dot{U}_r = \frac{d\,U_r}{d\,\Phi_i}\,\frac{d\,\Phi_i}{d\,t} = g_r^u\,\omega_i$$

$$\dot{V}_r = \frac{d\,V_r}{d\,\Phi_i}\,\frac{d\,\Phi_i}{d\,t} = g_r^v\,\omega_i$$

$$\dot{\Phi}_{pq} = \frac{d\,\Phi_{pq}}{d\,\Phi_i}\,\frac{d\,\Phi_i}{d\,t} = g_{pq}^{\Phi}\,\omega_i$$

$$\dot{U}_r = [\,g_p^u - g_{pq}^{\Phi}\,(x_r\,s_{pq} + y_r\,c_{pq})\,]\,\omega_i = g_r^u\,\omega_i$$

$$\therefore\ g_r^u = g_p^u - g_{pq}^{\Phi}\,(x_r\,s_{pq} + y_r\,c_{pq})$$

$$\dot{V}_r = [\,g_p^v + g_{pq}^{\Phi}\,(x_r\,c_{pq} - y_r\,s_{pq})\,]\,\omega_i = g_r^v\,\omega_i$$

$$\therefore\ g_r^v = g_p^v + g_{pq}^{\Phi}\,(x_r\,c_{pq} - y_r\,s_{pq})$$

Figure A.1 The Concept of Velocity Influence Coefficients

$$\bar{g}^v_{pq} = g^v_p + g^\phi_{pq}(\bar{x}_{pq}c_{pq} - \bar{y}_{pq}s_{pq})$$

into Eq. (A.1), gives

$$I^*_{pq} = m_{pq}[(g^u_p)^2 - 2g^u_p g^\phi_{pq}(\bar{x}_{pq}s_{pq} + \bar{y}_{pq}c_{pq}) + (g^v_p)^2 +$$

$$+ 2g^v_p g^\phi_{pq}(\bar{x}_{pq}c_{pq} - \bar{y}_{pq}s_{pq})] + m_{pq}(\bar{x}^2_{pq} + \bar{y}^2_{pq} + \bar{k}^2_{pq})(g^\phi_{pq})^2 \qquad \text{(A.2)}$$

Now, considering point "q," the following can be written

$$g^u_q = g^u_p - g^\phi_{pq}(a_{pq}s_{pq})$$

$$\therefore \qquad g^\phi_{pq}s_{pq} = (g^u_p - g^u_q) \div a_{pq} \qquad \text{(A.3)}$$

Similarly,

$$g^v_q = g^v_p + g^\phi_{pq}(a_{pq}c_{pq})$$

$$\therefore \qquad g^\phi_{pq}c_{pq} = (g^v_q - g^v_p) \div a_{pq} \qquad \text{(A.4)}$$

Squaring Eqs. (A.3) and (A.4), and combining them, yields

$$(g^\phi_{pq})^2 = [(g^u_p - g^u_q)^2 + (g^v_q - g^v_p)^2] \div a^2_{pq} \qquad \text{(A.5)}$$

Substituting Eqs. (A.3), (A.4) and (A.5) into Eq. (A.2), and re-grouping the terms, gives

$$I^*_{pq} = m_{pq}\{(g^u_p)^2 + (g^v_p)^2 + 2\bar{x}_{pq}[g^v_p(g^v_q - g^v_p) - g^u_p(g^u_p - g^u_q)]/a_{pq} -$$

$$- 2\bar{y}_{pq}[g^v_p(g^u_p - g^u_q) + g^u_p(g^v_q - g^v_p)]/a_{pq}\} +$$

$$+ m_{pq}(\bar{x}^2_{pq} + \bar{y}^2_{pq} + \bar{k}^2_{pq})[(g^u_p - g^u_q)^2 + (g^v_q - g^v_p)^2]/a^2_{pq}$$

Re-grouping the terms once more, yields

$$I^*_{pq} = [m_{pq}(a^2_{pq}-2a_{pq}\bar{x}_{pq}+\bar{x}^2_{pq}+\bar{y}^2_{pq}+\bar{k}^2_{pq})/a^2_{pq}][(g^u_p)^2+(g^v_p)^2] +$$
$$+ [m_{pq}(a_{pq}\bar{x}_{pq}-\bar{x}^2_{pq}-\bar{y}^2_{pq}-\bar{k}^2_{pq})/a^2_{pq}]2(g^u_pg^u_q+g^v_pg^v_q) +$$
$$+ (m_{pq}\bar{y}_{pq}/a_{pq})2(g^v_pg^u_q-g^u_pg^v_q) +$$
$$+ [m_{pq}(\bar{x}^2_{pq}+\bar{y}^2_{pq}+\bar{k}^2_{pq})/a^2_{pq}][(g^u_q)^2+(g^v_q)^2]$$

The input torque due to the inertia of link pq can be written as

$$^1T_{pq} = dKE_{pq}/d\phi_i = d(0.5I^*_{pq}\omega^2_i)/d\phi_i = 0.5[dI^*_{pq}/d\phi_i]\omega^2_i$$

$$\therefore \quad ^1T_{pq} = Y^3_{1pq}(g^u_ph^u_p+g^v_ph^v_p)\omega^2_i +$$
$$+ Y^3_{2pq}(g^u_ph^u_q+g^u_qh^u_p+g^v_ph^v_q+g^v_qh^v_p)\omega^2_i +$$
$$+ Y^3_{3pq}(g^v_ph^u_q+g^u_qh^v_p-g^u_ph^v_q-g^v_qh^u_p)\omega^2_i +$$
$$+ Y^3_{4pq}(g^u_qh^u_q+g^v_qh^v_q)\omega^2_i \qquad (A.6)$$

where Y^3_{ipq}(i = 1 to 4) are the same coefficients given in Eqs. (2.8.2), and h^u_p, h^v_p, h^u_q, and h^v_q are acceleration influence coefficients with the following property:

$$\ddot{U}_j = h^u_j\omega^2_i+g^u_j\alpha_i$$

and

$$\ddot{V}_j = h^v_j\omega^2_i+g^v_j\alpha_i \qquad ; \; j = p,q$$

where α_i = angular acceleration of input link = 0. Therefore, Eq. (A.6) can be rewritten as

$$^{1}T_{pq} = Y^{3}_{1pq}D^{6}_{1pq} + Y^{3}_{2pq}D^{6}_{2pq} + Y^{3}_{3pq}D^{6}_{3pq} + Y^{3}_{4pq}D^{6}_{4pq}$$

where

$$D^{6}_{1pq} = (\dot{U}_p\ddot{U}_p + \dot{V}_p\ddot{V}_p) \div \omega_i$$

$$D^{6}_{2pq} = [(\dot{U}_p\ddot{U}_q + \dot{V}_p\ddot{V}_q) + (\dot{U}_q\ddot{U}_p + \dot{V}_q\ddot{V}_p)] \div \omega_i$$

$$D^{6}_{3pq} = [-(\dot{U}_p\ddot{V}_q - \dot{V}_p\ddot{U}_q) + (\dot{U}_q\ddot{V}_p - \dot{V}_q\ddot{U}_p)] \div \omega_i$$

$$D^{6}_{4pq} = (\dot{U}_q\ddot{U}_q + \dot{V}_q\ddot{V}_q) \div \omega_i$$

APPENDIX B
EFFECTS OF A GROUNDED LINK ON SHAKING MOMENT EXPRESSION

From Fig. B.1, the following equation can be written

$$^{s}M_{12} = \underline{r}_1 \times \underline{F}_{12} \cdot \underline{k} + I_{12}\ddot{\phi}_{12}$$

Substiting $\underline{F}_{12} = m(\bar{x}\ddot{\underline{e}}_x + \bar{y}\ddot{\underline{e}}_y)$[1] into the above equation, results in

$$^{s}M_{12} = m\bar{x}[\underline{r}_1 \times \ddot{\underline{e}}_x \cdot \underline{k}] + m\bar{y}[\underline{r}_1 \times \ddot{\underline{e}}_y \cdot \underline{k}] + m(\bar{x}^2 + \bar{y}^2 + \bar{k}^2)[\ddot{\phi}_{12}] \qquad \text{(B.1)}$$

Thus, it can be concluded that the three kinematic coefficients inside the brackets of Eq. (B.1) are, in general, linearly independent, and so are the three coefficients multiplying the brackets. Clearly, if $\ddot{\phi}_{12}$ =

1 For simplification, the subscripts "12" will be dropped out of m_{12}, $\bar{x}_{12}$, $\bar{y}_{12}$, $\bar{k}_{12}$, and a_{12}, since link 12 is the only one addressed here.

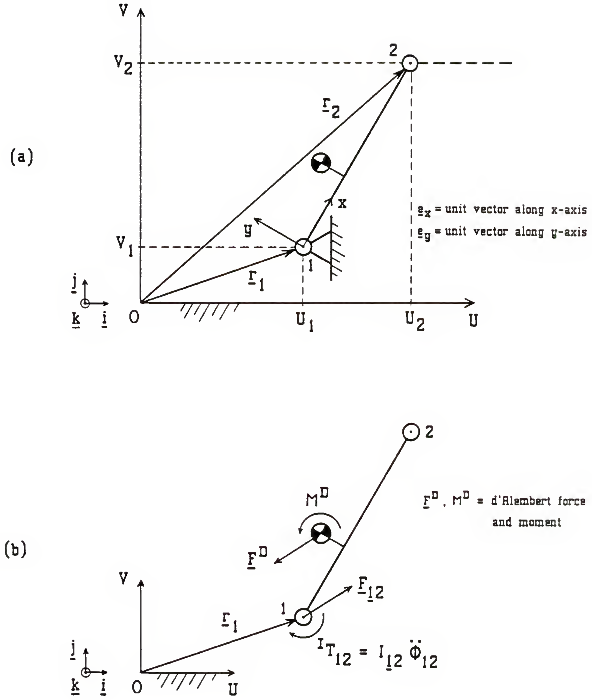


Figure B.1 A Grounded Link
a) Diagram of an Input Link 12
b) Free-Body Diagram of Input Link 12 When All the Other Links are Considered Massless

constant, the third term which corresponds to the input torque contribution to shaking moment vanishes, and link 12 will allow only two specifications for the latter dynamic property. If "O" is located at joint 1 ($\underline{r}_1 = \underline{0}$), then the first two terms go to zero ($\underline{F}_{12}$ does not contribute any moment) and only one shaking moment specification can be made for link 12. Of course, no specification can be made if the two cases above occur simultaneously. All this is in agreement with section 2.8. To expose even further the relationships between Eq. (B.1) and the formulation given in section 2.8, the following rearrangement of Eq. (B.1) is appropriate:

$$
\begin{aligned}
{}^{S}M_{12} &= m\bar{x}[\underline{r}_1 \times a^{-1}\ddot{\underline{r}}_2 \cdot \underline{k}] + m\bar{y}[\underline{r}_1 \cdot a^{-1}\ddot{\underline{r}}_2] + m(\bar{x}^2+\bar{y}^2+\bar{k}^2)[\ddot{\phi}_{12}] \\
&= (m\bar{x}/a)[U_1\ddot{V}_2 - V_1\ddot{U}_2] + (m\bar{y}/a)[U_1\ddot{U}_2 + V_1\ddot{V}_2] + \\
&\quad + m(\bar{x}^2+\bar{y}^2+\bar{k}^2)a^{-2}[a^2\ddot{\phi}_{12}]
\end{aligned}
$$

Everything stated in the previous paragraph will still hold if the above equation is modified as follows:

$$
\begin{aligned}
{}^{S}M_{12} &= \{m\bar{x}a^{-1} - m(\bar{x}^2+\bar{y}^2+\bar{k}^2)a^{-2}\}[U_1\ddot{V}_2 - V_1\ddot{U}_2] + \\
&\quad + (m\bar{y}/a)[U_1\ddot{U}_2 + V_1\ddot{V}_2] + m(\bar{x}^2+\bar{y}^2+\bar{k}^2)a^{-2}[a^2\ddot{\phi}_{12} + U_1\ddot{V}_2 - V_1\ddot{U}_2] \\
&= m(a\bar{x} - \bar{x}^2 - \bar{y}^2 - \bar{k}^2)a^{-2}[U_1\ddot{V}_2 - V_1\ddot{U}_2] + (m\bar{y}/a)[U_1\ddot{U}_2 + V_1\ddot{V}_2] +
\end{aligned}
$$

$$+ m(\bar{x}^2+\bar{y}^2+\bar{k}^2)a^{-2}[U_2\ddot{V}_2-V_2\ddot{U}_2]$$

$$= Y^3_{212}[D^4_{212}]+Y^3_{312}[D^4_{312}]+Y^3_{412}[D^4_{412}]$$

A similar analysis can be carried out for a grounded link that is not the input link and the same conclusions achieved.

APPENDIX C
ELIMINATION OF LINEAR DEPENDENCIES FOR CASES INVOLVING PRISMATIC JOINTS

The formulations derived in sections 2.4, 2.5, and 2.6 for links with p-ϕ motion specification were left with a very attractive configuration. Hence, one can easily comprehend and identify the linear dependences indicated in Fig. C.1 excepting, probably, case 4 for shaking moment and input torque whose proof is given below.

n=3 and m=4 (shaking moment):

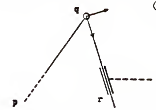
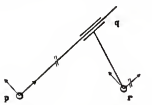
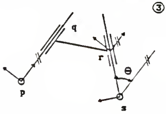
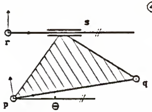
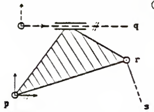
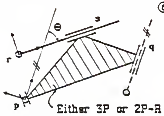
$$(D^4_{1pq}-D^4_{2pq}+D^4_{4pq})\div a^2_{pq} = [(U_p\ddot{V}_p-V_p\ddot{U}_p)-(U_p\ddot{V}_q-V_p\ddot{U}_q) - (U_q\ddot{V}_p-V_q\ddot{U}_p)+(U_q\ddot{V}_q-V_q\ddot{U}_q)]\div a^2_{pq}$$

$$= [(\underline{r}_p\times\ddot{\underline{r}}_p)-(\underline{r}_p\times\ddot{\underline{r}}_q)-(\underline{r}_q\times\ddot{\underline{r}}_p)+(\underline{r}_q\times\ddot{\underline{r}}_q)].\underline{k}\div a^2_{pq}$$

where $\underline{r}_p$ and $\underline{r}_q$ are the position vectors of points "p" and "q" with respect to the global coordinate system. Continuing,

$$(D^4_{1pq}-D^4_{2pq}+D^4_{4pq})\div a^2_{pq} = [\underline{r}_p\times(\ddot{\underline{r}}_p-\ddot{\underline{r}}_q)-\underline{r}_q\times(\ddot{\underline{r}}_p-\ddot{\underline{r}}_q)].\underline{k}\div a^2_{pq}$$

$$= [(\underline{r}_p-\underline{r}_q)\times(\ddot{\underline{r}}_p-\ddot{\underline{r}}_q)].\underline{k}\div a^2_{pq}$$

<table>
<tr><td>①
</td><td>$[D^m_{4pq} = D^m_{1qp} = D^n_{1qp}] = D^n_{1qr}$; $n=1,3,5$; $m=n+1$</td></tr>
<tr><td>②
</td><td>$D^1_{2pq} = D^1_{2rq}$
$D^n_{4pq} = D^n_{4rq}$; $n=3,5$</td></tr>
<tr><td>③
</td><td>$D^1_{2pq} = D^1_{2rq} = (\cos\Theta + i\sin\Theta)\, D^1_{2sr}$
$D^n_{4pq} = D^n_{4rq} = D^n_{4sr}$: $n=3,5$</td></tr>
<tr><td>④
</td><td>$D^1_{2rs} = \frac{1}{a_{pq}}(\cos\Theta + i\sin\Theta)(D^2_{4pq} - D^2_{1pq})$
$D^n_{4rs} = (D^m_{1pq} - D^m_{2pq} + D^m_{4pq}) \div a^2_{pq}$: $n=3,5$: $m=n+1$</td></tr>
<tr><td>⑤
</td><td>$[D^2_{1rs} = D^2_{4sr} = D^1_{1rs}] = D^1_{1pq} + (x_r + i\, y_r)\, D^1_{2pq}$
$[D^m_{1rs} = D^m_{4sr} = D^n_{1rs}] = D^n_{1pq} + x_r D^n_{2pq} + y_r D^n_{3pq} + (x_r^2 + y_r^2)\, D^n_{4pq}$
$n=3,5$: $m=n+1$</td></tr>
<tr><td>⑥
</td><td>$D^1_{2rs} = (\cos\Theta + i\sin\Theta)\, D^1_{2pq}$
$D^n_{4rs} = D^n_{4pq}$: $n=3,5$</td></tr>
</table>

Note 1: Adapted from Table 3.5.1 and Fig. B.1 of Ref. [1].

Note 2: $n=1$ & $m=2$ for ${}^S\underline{F}$. $n=3$ & $m=4$ for ${}^S M$. $n=5$ & $m=6$ for ${}^I T$.

Figure C.1 Linear Dependencies Involving Prismatic Joints

$$= \{(\underline{r}_p - \underline{r}_q) \times [\ddot{\phi}_{pq}\underline{k} \times (\underline{r}_p - \underline{r}_q) - \dot{\phi}_{pq}^2 (\underline{r}_p - \underline{r}_q)]\} \cdot \underline{k} \div a_{pq}^2$$

$$= \ddot{\phi}_{pq} = D_{4rs}^3 \qquad \text{Q.E.D.}$$

The proof for n=5 and m=6 (input torque) is very similar to the previous one. One just needs to recognize from Fig. 2.19 that

$$D_{1pq}^6 = (\dot{\underline{r}}_p \cdot \ddot{\underline{r}}_p) \div \omega_i$$

$$D_{2pq}^6 = [(\dot{\underline{r}}_p \cdot \ddot{\underline{r}}_q) + (\dot{\underline{r}}_q \cdot \ddot{\underline{r}}_p)] \div \omega_i$$

and

$$D_{4pq}^6 = (\dot{\underline{r}}_q \cdot \ddot{\underline{r}}_q) \div \omega_i$$

APPENDIX D
PROOF THAT THE MASS OF A LINK GROUNDED BY A PIN JOINT NEED NOT BE CONSIDERED INDIVIDUALLY WHEN CALCULATING REACTION FORCES

The shaking force acting on a linkage due to the inertia of one of its links corresponds exactly to the resultant force acting on that link. Therefore, Eq. (2.4.1) can be applied to link pq in Fig. D.1 in the following way:

$$m_{pq}\ddot{\underline{r}}_G = Y^1_{2pq}D^1_{2pq}+Y^1_{3pq}D^1_{3pq} \quad ; \ (D^1_{1pq} = 0)$$

$$= m_{pq}\bar{x}_{pq}[\ddot{\phi}_{pq}(-s_{pq}+ic_{pq})-\dot{\phi}^2_{pq}(c_{pq}+is_{pq})] +$$

$$+ m_{pq}\bar{y}_{pq}[-\ddot{\phi}_{pq}(c_{pq}+is_{pq})+\dot{\phi}^2_{pq}(s_{pq}-ic_{pq})]$$

Rearranging and considering separately the two global components of the resultant force above, yields

ΣF_{pqu} = summation of the U-components of all external and reaction forces acting on link pq

$$= x^o_{pq}a_{pq}(-\ddot{\phi}_{pq}s_{pq}-\dot{\phi}^2_{pq}c_{pq})+y^o_{pq}a_{pq}(-\ddot{\phi}_{pq}c_{pq}+\dot{\phi}^2_{pq}s_{pq}) \quad (D.1)$$

and

$$\Sigma F_{pqv} = x^o_{pq}a_{pq}(\ddot{\phi}_{pq}c_{pq}-\dot{\phi}^2_{pq}s_{pq})+y^o_{pq}a_{pq}(-\ddot{\phi}_{pq}s_{pq}-\dot{\phi}^2_{pq}c_{pq}) \quad (D.2)$$

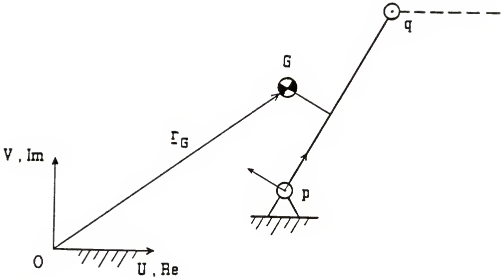


Figure D.1 A Grounded Link That Fulfills Rule 1 of Section 2.2

The third equation of dynamic equilibrium to be considered in the calculation of reaction forces can be the following:

$$\Sigma M_{pq} = \text{summation of all the moments about point "p"}$$

$$= m_{pq}(\bar{x}^2_{pq}+\bar{y}^2_{pq}+\bar{k}^2_{pq})\ddot{\phi}_{pq} = k^o_{pq}a^2_{pq}\ddot{\phi}_{pq} \qquad (D.3)$$

Equations (D.1) to (D.3) do not contain m_{pq} explicitly because Rule 1 of section 2.2 has been satisfied.

APPENDIX E
APL FUNCTIONS COMPOSING THE "GRAPHICS SYSTEM"

(The majority of the functions in this appendix have been adapted from functions created by Matthew B. Reischer.)

```
    ∇ GFACT N
[1]    ⍝ GFACT - MAKES REGION N THE ACTIVE DRAWING REGION .
[2]    ⍝ N     - NUMBER OF THE REGION TO BE ACTIVATED .
[3]   GVARG←,GVRDT[GVRDT[;1]⍳N;]
    ∇

    ∇ C GFARC RSΔI;NSEG;T;CO;S;START;V;M
[1]    ⍝ GFARC    - DRAWS AN ARC IN THE ACTIVE REGION
[2]    ⍝ C        - POSITION OF THE CENTER IN LOG. COORDS
[3]    ⍝ RSΔI[1] - RADIUS OF THE ARC IN LOGICAL COORDS
[4]    ⍝ RSΔI[2] - STARTING ANGLE IN DEGREE (POS. OR NEG.)
[5]    ⍝ RSΔI[3] - ANGLE DEFINED BY THE ARC (POS. OR NEG.)
[6]    ⍝ RSΔI[4] - NO. OF SEGMENTS IN ARC (OPTIONAL)
[7]   NSEG←90
[8]   →(3=⍴RSΔI)/JUMP
[9]   NSEG←RSΔI[4]
[10] JUMP:T←(2 2)⍴CO,(-S),(S←1○T),CO←2○T←(RSΔI[3]×○÷180)÷NSEG
[11]  V←(RSΔI[1]×(2○START),1○START←RSΔI[2]×○÷180),(2,NSEG)⍴0
[12]  M←0
[13]  ⍎5↓(14×NSEG+1)⍴'←T+.×V[;M←M+1]'
[14]  V←V+(⍴V)⍴((NSEG+1)⍴C[1]),(NSEG+1)⍴C[2]
[15]  (0,NSEG⍴1)GFVABS⍎((11⍴∧/1 1=GVARG[10 11])/'⌊((⍴V)⍴.5)+')
      ,'V'
    ∇

    ∇ C GFCIRC R;T;S;CO;M;V;NSEG
[1]    ⍝ GFCIRC - DRAWS A CIRCLE IN THE ACTIVE REGION
[2]    ⍝ C      - POSITION OF THE CENTER IN LOGICAL COORDS
[3]    ⍝ R      - RADIUS OF THE CIRCLE IN LOGICAL COORDS;
[4]    ⍝          IF ⍴R=2, R[2]= NO. OF SEGMENTS IN CIRCLE
[5]    ⍝ T      - ROTATION MATRIX
[6]    ⍝ NSEG   - NUMBER OF SEGMENTS IN CIRCLE
[7]   NSEG←120
[8]   →(2≠⍴,R)/START
[9]   NSEG←¯1↑R
[10]  R←(⍳0)⍴1↑R
[11] START:T←(2,2)⍴CO,S,(-S←1○T),CO←2○T←(○2)÷NSEG
[12]  V←(0,R),(2,NSEG)⍴0
[13]  M←0
[14]  ⍎5↓(14×NSEG+1)⍴'←T+.×V[;M←M+1]'
[15]  V←V+(⍴V)⍴((NSEG+1)⍴C[1]),(NSEG+1)⍴C[2]
[16]  (0,NSEG⍴1)GFVABS⍎((11⍴∧/1 1=GVARG[10 11])/'⌊((⍴V)⍴.5)+')
      ,'V'
    ∇

    ∇ GFCLA;T
[1]    ⍝  CLEARS SCREEN AND LEAVES TERMINAL IN ALPHA MODE .
[2]   ⎕ARBOUT 27 12
[3]   T←⎕DL 1
    ∇
```

```
    ∇ GFCLG;T
[1]    ⍝ CLEARS SCREEN AND LEAVES TERMINAL IN GRAPHICS MODE .
[2]   ⎕ARBOUT 27 12 29
[3]   T←⎕DL .5
    ∇

    ∇ P1 GFDALINE P2;A;B;P1P2PH;R12;MAG;R12UNIT;RATIO;INT;V1;V
      2;Δ;PV;IB;I;KEEP
[1]    ⍝ GFDALINE - DRAWS AN INTERMITENT LINE SEGMENT FROM P1
      TO P2(LOG.
[2]   ⍝             COORDS.) IN THE CURRENTLY ACTIVE REGION.
[3]    ⍝ DASHTYPE - GLOBAL VARIABLE THAT DEFINES THE TYPE OF T
      HE LINE SEG:
[4]    ⍝              DASHTYPE=1 → DOTTED LINE
[5]    ⍝                       =2 → SHORT-DASHED LINE
[6]    ⍝                       =3 → MEDIUM-DASHED LINE
[7]    ⍝                       =4 → LONG-DASHED LINE
[8]    ⍝                       =5 → DOT-DASHED LINE
[9]   ⍎'P1P2PH←',((5⍴∨/1 1 0 0≠KEEP←GVARG[10 11 12 13])/'GFLTP
      '),' P1,[1.1]P2'
[10]  R12UNIT←R12÷MAG←(+/(R12←-−/P1P2PH)*2)*.5
[11]  I←0
[12]  GVARG[10 11 12 13]←1 1 0 0
[13] AGAIN:A←1 6 8 10 12 1[DASHTYPE+I]
[14]  B←6 4 5 5 12 5[DASHTYPE+I]
[15]  INT←⌊RATIO←MAG÷A+B
[16]  V1←⌊.5++\P1P2PH[;1],Δ←⍉(INT,2)⍴R12UNIT×A+B
[17]  V2←⌊.5++\(P1P2PH[;1]+A×R12UNIT),Δ
[18]  PV←(0 ¯1↓(2,2×INT+1)⍴V1,[2.1]V2),P1P2PH[;2]
[19]  IB←(2×INT+1)⍴0 1
[20]  IB[1+2×INT]←(RATIO-INT)≤.3
[21]  IB GFVABS⍎((12⍴∧/1 1=GVARG[10 11])/'⌊((⍴PV)⍴.5)+'),'PV'
[22]  →AGAIN×⍳I←(I=0)∧DASHTYPE=5
[23]  GVARG[10 11 12 13]←KEEP
    ∇

    ∇ LL GFDARECT UR
[1]   ⍝ GFDARECT - DRAWS A RECTANGLE WITH INTERMITENT LINE SEG
      MENTS.
[2]   ⍝ DASHTYPE - GLOBAL VARIABLE DEFINING THE TYPE OF THE LI
      NE SEG.
[3]   ⍝             (SEE COMMENTS IN 'GFDALINE' AND 'GFRECT')
[4]   LL GFDALINE UR[1],LL[2]
[5]   (UR[1],LL[2])GFDALINE UR
[6]   UR GFDALINE LL[1],UR[2]
[7]   (LL[1],UR[2])GFDALINE LL
    ∇
```

```
    ∇ GFDREG R
[1]    ⍝ GFDREG - DELETES REGION(S) FROM THE REGION DEFINITION
       TABLE .
[2]    ⍝ R      - NUMERIC SCALAR(OR NUM VECTOR) SPECIFYING THE
       REGION(S) TO
[3]    ⍝          BE DELETED FROM GVRDT.
[4]   GVRDT←(~GVRDT[;1]∊R)/[1]GVRDT
    ∇

    ∇ GFDRW P;B
[1]   ⍝ GFDRW - DRAWS A LINE SEGMENT FROM CURRENT POSITION
[2]   ⍝         OF GRAPHIC CURSOR TO P. THE TERMINAL MUST
[3]    ⍝         ALREADY BE IN GRAPHICS MODE, AND WILL BE LEFT

[4]    ⍝         IN GRAPHICS MODE.
[5]   ⍝ P     - A POINT EXPRESSED IN LOGICAL COORDS. IN THE
[6]    ⍝         ACTIVE REGION TO WHICH THE LINE SEGMENT IS TO
      BE DRAWN .
[7]   1 GFVABS ⍎((11ρ∧/1 1 0 0=GVARG[10 11 12 13])/'⌊((ρP)ρ.5)
      +'),'P'
    ∇

    ∇ Z←GFGCUR;⎕TX;A
[1]    ⍝ GFGCUR - BRINGS UP THE CROSSHAIR CURSORS
[2]    ⍝ Z      - Z[1] X-COORD
[3]    ⍝          Z[2] Y-COORD (LOGICAL)
[4]    ⍝          Z[3] ⎕AV⍳(WHATEVER KEY YOU HIT)
[5]   ⎕TX←'⎕ARBOUT 7⋄→5'
[6]   ⎕ARBOUT 29 27 26
[7]   Z←⌽32 64+⌽32⊥⍉2 2ρ(¯1+⎕AV⍳1↓A←⍞)-32 96 32 64
[8]   Z←(GFPTL Z),⎕AV⍳1↑A
    ∇

    ∇ P1 GFLINE P2;V
[1]    ⍝ GFLINE - DRAWS A LINE SEGMENT FROM P1 TO P2 . THE TER
      MINAL MUST
[2]    ⍝          ALREADY BE IN GRAPHICS MODE, AND WILL BE LEF
      T IN GRAPHICS MODE .
[3]    ⍝ P1,P2  - POINTS EXPRESSED IN LOGICAL COORDINATES
[4]    ⍝          IN THE CURRENTLY ACTIVE REGION .
[5]   0 1 GFVABS⍎((11ρ∧/1 1 0 0=GVARG[10 11 12 13])/'⌊((ρV)ρ.5
      )+'),'V←P1,[1.5]P2'
    ∇

    ∇ PP←GFLTP LP
[1]    ⍝ GFLTP - CONVERTS THE LOGICAL COORDINATES OF THE POINT
      (S) SPECIFYIED
[2]    ⍝         BY LP TO PHYSICAL SCREEN COORDINATES .
[3]    ⍝ LP    - A 2×N ARRAY OF LOGICAL COORDS OF N POINTS IN
      THE CURRENTLY ACTIVE REGION .
[4]    ⍝         IF N=1 EITHER ρLP=2 1 OR ρLP=2 WILL BE OK .
[5]   ⍝ PP    - CONVERTED PHYSICAL SCREEN COORDS. (ρPP=ρLP) .
[6]   →L×⍳1=ρρLP
```

```
[7]   PP←(⌊.5+GVARG[12]+LP[1;]×GVARG[10]),[.5]⌊.5+GVARG[13]+LP
      [2;]×GVARG[11]
[8]   →0
[9]  L:PP←⌊.5+(¯2↑GVARG)+LP×GVARG[10 11]
    ∇

    ∇ GFMOV P
[1]    ⍝ GFMOV - MOVES THE CURSOR INVISIBLY TO THE POINT P .
[2]    ⍝         THE TERMINAL WILL BE LEFT IN GRAPHICS MODE .
[3]    ⍝ P     - A POINT (P[1],P[2]) EXPRESSED IN LOGICAL
[4]    ⍝         COORDINATES IN THE CURRENTLY ACTIVE REGION .
[5]   0 GFVABS⍎((11⍴∧/1 1 0 0=GVARG[10 11 12 13])/'⌊((⍴P)⍴.5)+
      '),'P'
    ∇

    ∇ GFPBUF RS
[1]    ⍝ GFPBUF - PURGES THE WHOLE BUFFER IF 0 ∊ RS, OTHERWISE

[2]   ⍝          ERASES ONLY THE COLUMNS OF GVBUF CORRESP.
[3]    ⍝          TO THE REGIONS INDICATED BY RS .
[4]    ⍝ RS     - NUMERIC SCALAR OR NUMERIC VECTOR .
[5]   →LB×⍳~0∊RS
[6]   GVBUF←4 1⍴0
[7]   →0
[8]  LB:GVBUF←(~GVBUF[4;]∊RS)/GVBUF
    ∇

    ∇ L←GFPTL P
[1]    ⍝ GFPTL - CONVERTS PHYSICAL COORDINATES IN P TO ITS
[2]   ⍝         CORRESP. LOG. COORDS. IN THE ACTIVE REGION.
[3]    ⍝ P     - A 2×N ARRAY OF PHYSICAL COORDS. OF N POINTS O
      F THE SCREEN.
[4]    ⍝        IF N=1, EITHER ⍴P=2 1 OR ⍴P=2 WILL BE OK.
[5]    ⍝ L     - CONVERTED LOGICAL COORDINATES (⍴L=⍴P).
[6]   →((⍴⍴P)=2)/LINE
[7]   L←GVARG[6 7]+(P-GVARG[2 3])÷GVARG[10 11]
[8]   →0
[9]  LINE:L←GVARG[6]+(P[1;]-GVARG[2])÷GVARG[10]
[10]  L←L,[.1]GVARG[7]+(P[2;]-GVARG[3])÷GVARG[11]
    ∇

    ∇ Z←GFRBUF RS;Q;T;PV;IB
[1]    ⍝ GFTBUF - TRANSMITS THE WHOLE BUFFER(GVBUF) IF 0 ∊ RS
[2]   ⍝          OTHERWISE TRANSMITS ONLY REGIONS INDICATED
[3]    ⍝          BY RS AND FOUND IN GVBUF .
[4]    ⍝ RS     - NUMERIC SCALAR OR NUMERIC VECTOR .
[5]   →L1×⍳~0∊RS
[6]   Q←(¯1+¯1↑⍴GVBUF)⍴1
[7]   →L1+1
[8]  L1:Q←1↓GVBUF[4;]∊RS
[9]   IB←Q/1↓GVBUF[1;]
[10]  PV←Q/0 1↓GVBUF[2 3;]
[11]  T←,⍉(29×~IB),[1]Y[;1+PV[2;]],[1]X[;1+PV[1;]]
[12]  Z←(T≠0)/T
[13]  GVCUR←,2 ¯1↑PV
    ∇
```

```
    ∇ LL GFRECT UR;I
[1]    ⍝ GFRECT - DRAWS A RECTANGLE .
[2]    ⍝ LL     - (X,Y) COORDINATE OF THE LOWER LEFT CORNER .
[3]   ⍝ UR     - (X,Y) COORDINATE OF THE UPPER RIGHT CORNER.
[4]   ⍝           BOTH COORD'S EXPRESSED IN LOGICAL COORD'S
[5]   ⍝           IN THE CURRENTLY ACTIVE REGION .
[6]   →LB×⍳(1=ρρLL)∨3=+/ρLL
[7]   (Iρ0 1 1 1 1)GFVABS⍉((I←5×¯1↑ρLL),2)ρ(⍉LL),LL[1;],UR[2;]
      ,(⍉UR),UR[1;],LL[2;],⍉LL
[8]   →0
[9]  LB:UR←,UR
[10]  0 1 1 1 1 GFVABS LL,(LL[1],UR[2]),UR,(UR[1],LL[2]),[1.5]
      LL←,LL
    ∇


    ∇ N GFREG C
[1]    ⍝ GFREG  - ASSIGNS A NUMBER AND PHYSICAL CO'ORDS TO A R
      EGION .
[2]    ⍝ N      - REGION NUMBER .
[3]    ⍝ C      - PHYSICAL CO'ORDS (LLX,LLY,URX,URY) .
[4]    ⍝ GVRDT  - REGION DEFINITION TABLE(GLOBAL):
[5]    ⍝       (NUMBER,LLXP,LLYP,URXP,URYP,LLXL,LLYL,URXL,URYL,S
      CLX,SCLY,ORGXP,ORGYP)
[6]   →1+⎕LC[1]+2×N∊GVRDT[;1]
[7]   GVRDT←GVRDT,[1](ρGVRDT)[2]↑N,C
[8]   →0
[9]   GVRDT[GVRDT[;1]⍳N;1+⍳4]←C
    ∇


    ∇ N GFSCL T
[1]   ⍝ GFSCL - MODIFIES THE REG. DEFINITION TBLE, GFRDT, BY
[2]   ⍝         ASSOCIATING LOG. COORDS. WITH THE REGION N
[3]    ⍝         INITIALIZED BY GFREG.
[4]   ⍝ T     - LOGICAL REGION COORDS. (LLX,LLY,URX,URY).
[5]   N←GVRDT[;1]⍳N
[6]   GVRDT[N;5+⍳6]←T,((GVRDT[N;4]-GVRDT[N;2])÷T[3]-T[1]),(GVR
      DT[N;5]-GVRDT[N;3])÷T[4]-T[2]
[7]   GVRDT[N;12 13]←GVRDT[N;2 3]-T[1 2]×GVRDT[N;10 11]
    ∇


    ∇ GFSTUP
[1]    ⍝ INITIALIZES REGION DEFINITION TABLE(GVRDT) BY
[2]    ⍝ ASSIGNING TO IT THE BASE REGION(REGION 1): FULL
[3]    ⍝ SCREEN; BOTH LOGICAL AND PHISICAL ORIGINS AT
[4]    ⍝ THE LL CORNER; BOTH X AND Y SCALE FACTORS = 1 .
[5]    ⍝ INITIALIZES BUFFER(GVBUF) AND SETS THE TRANSMISSION
[6]    ⍝ CODE(TRC) TO AMEND AND TRANSMIT .
[7]   GVRDT←(1 13)ρ1,0,0,1023,779,0,0,1023,779,1,1,0,0
[8]   GVBUF←(4,1)ρ0
[9]   TRC←'AT'
    ∇
```

```
    ∇ GFTBUF RS;Q;T;PV;IB
[1]    ⍝ GFTBUF - TRANSMITS THE WHOLE BUFFER(GVBUF) IF 0 ∊ RS
[2]    ⍝          OTHERWISE TRANS. ONLY THE REGIONS INDICATED
[3]    ⍝            BY RS AND FOUND IN GVBUF .
[4]    ⍝ RS      - NUMERIC SCALAR OR NUMERIC VECTOR .
[5]    →L1×⍳~0∊RS
[6]    Q←(¯1+¯1↑⍴GVBUF)⍴1
[7]    →L1+1
[8]   L1:Q←1↓GVBUF[4;]∊RS
[9]    IB←Q/1↓GVBUF[1;]
[10]   PV←Q/0 1↓GVBUF[2 3;]
[11]   T←,⍉(29×~IB),[1]Y[;1+PV[2;]],[1]X[;1+PV[1;]]
[12]   ⎕ARBOUT(T≠0)/T
[13]   GVCUR←,2 ¯1↑PV
    ∇

    ∇ P GFTXT TEXT
[1]    ⍝ MOVES CURSOR TO P[1 2] (LOG.COORDS. OF A POINT IN THE
       ACTIVE REG.).
[2]    ⍝ IF ⍴P=3, BACKSPACES THE NUM. OF TIMES SPECIFIED BY P[
      3]. THEN
[3]    ⍝ DISPLAYS THE CHAR. VECTOR TEXT.
[4]    GFMOV 2↑P
[5]    ⎕ARBOUT 31
[6]    →(2+⎕LC[1])×⍳2=⍴P
[7]    ⎕ARBOUT P[3]⍴8
[8]    TEXT
    ∇

    ∇ IB GFVABS XY;T;PV
[1]    ⍝ GFVABS - TAKES AN X,Y VECTOR OF COORDS AND DRAWS THEM
       AS ABSOLUTE
[2]    ⍝           COORDS IN THE ACTIVE REGION.
[3]    ⍝ XY      - ARRAY OF ABSOLUTE COORDS, ⍴XY=(2,N) .
[4]    ⍝ IB      - CONNECTIVITY VECTOR; IB[I] = 0: MOVE CURSOR
      INVISIBLY TO XY[;I]
[5]    ⍝                                   = 1: DRAW FROM CU
      RRENT CURSOR POSITION TO XY[;I]
[6]    →2+(1↑⎕LC)×⍳2=⍴⍴⍎'PV←',((5⍴∨/1 1 0 0≠GVARG[10 11 12 13])
      /'GFLTP'),' XY'
[7]    PV←2 1⍴PV
[8]    →ER×⍳∨/((⌈/PV)>1023,779),(⌊/PV)<0
[9]    GVCUR←,2 ¯1↑PV
[10]   →(~'A'∊TRC)/2+1↑⎕LC
[11]   GVBUF←GVBUF,IB,[1]PV,[1](⍴,IB)⍴GVARG[1]
[12]   →(~'T'∊TRC)/0
[13]   T←,⍉(29×~IB),[1]Y[;1+PV[2;]],[1]X[;1+PV[1;]]
[14]   ⎕ARBOUT(T≠0)/T
[15]   →0
[16]  ER:⎕ARBOUT 13
[17]   'GRAPHICS ERROR: ATTEMPT TO DRAW OFF OF THE SCREEN'
[18]   )SI
[19]   →
    ∇
```

```
     ∇ IB GFVINC XY;T;⎕IO;PV
[1]     ⍝ GFVINC - TAKES AN X,Y VECTOR OF COORDS AND DRAWS THEM
        RELATIVE
[2]    ⍝           TO THE CURRENT POS. OF THE GRAPHICS CURSOR.
[3]     ⍝ XY      - ARRAY OF RELATIVE COORDS, ⍴XY=(2,N)
[4]     ⍝ IB      - CONNECTIVITY OF LINES, 0: MOVE CURSOR INVISI
       BLY TO XY
[5]     ⍝                                  1: DRAW FROM CURRENT
       CURSOR POSITION TO XY
[6]    →2+(1↑⎕LC)×⍳2=⍴⍴PV←0 1↓+\(2 1⍴GVCUR),⍎((5⍴∨/1 1 0 0≠GVAR
       G[10 11 12 13])/'GFLTP'),' XY'
[7]    PV←2 1⍴PV
[8]    →ER×⍳∨/(([/PV)>1023,779),(⌊/PV)<0
[9]    GVCUR←,2 ¯1↑PV
[10]   →(~'A'∊TRC)/2+1↑⎕LC
[11]   GVBUF←GVBUF,IB,[1]PV,[1](⍴,IB)⍴GVARG[1]
[12]   →(~'T'∊TRC)/0
[13]   T←,⍉(29×~IB),[1]Y[;1+PV[2;]],[1]X[;1+PV[1;]]
[14]   ⎕ARBOUT(T≠0)/T
[15]   →0
[16] ER:⎕ARBOUT 13
[17]   'GRAPHICS ERROR: ATTEMPT TO DRAW OFF OF THE SCREEN'
[18]   )SI
[19]   →
     ∇

     ∇ GFVP NS;I;II
[1]     ⍝ DRAWS VIEWPORT BOUNDARY ASSOCIATED TO THE REGION(S) I
       NDICATED BY NS(SCALAR
[2]     ⍝ OR VECTOR BUT ALWAYS NUMERIC).
[3]    NS←(1,I←⍴,NS)⍴NS
[4]    GFACT II←(⍳0)⍴NS[1;I]
[5]    (GVRDT[GVRDT[;1]⍳II;6 7])GFRECT GVRDT[GVRDT[;1]⍳II;8 9]
[6]    →¯2+1↑⎕LC×0≠I←I-1
     ∇
```

APPENDIX F
APL FUNCTIONS COMPOSING THE "APPLICATION PROGRAM"

```
    ∇ PHI←X ARG Y;Z
[1]    ⍝ CALCULATES ANGLE(S) BETWEEN X,Y VECTOR(S) AND X-AXIS.

[2]   PHI←((○.5)×Z××Y)+(~Z)×¯3○Y÷X+Z←X=0
[3]   PHI←PHI+○2×0>PHI←PHI+(X<0)×(○1)×(×Y)+Y=0
    ∇



    ∇ Z←BASICMP;L;MUV;C4;CXY
[1]   ⍝ TRANSFORMS LUMPED MASS PARAMETERS
[2]   ⍝ INTO BASIC MASS PARAMETERS
[3]   ΔM12X←M12MIN,M12MAX⋄ Z←0
[5]   →(LB2,LB1,⎕LC[1]+1)[2+×XO12]
[6]   ΔM12X←(¯1+2×(M12MIN×X12MAX)>L)↑M12MIN,(L←XO12×A12)÷X12MA
      X
[7]   ΔM12X←ΔM12X,⍎8↑(8×(M12MAX×X12MIN)>L)⌽'M12MAX  L÷X12MIN'
[8]   →LB1
[9]  LB2:ΔM12X←(¯1+2×(M12MIN×X12MIN)≤L)↑M12MIN,(L←XO12×A12)÷X1
      2MIN
[10]  ΔM12X←ΔM12X,⍎8↑(8×(M12MAX×X12MAX)≤L)⌽'M12MAX  L÷X12MAX'
[11] LB1:ΔM12Y←M12MIN,M12MAX
[12]  →(LB4,LB3,⎕LC[1]+1)[2+×YO12]
[13]  ΔM12Y←(¯1+2×(M12MIN×Y12MAX)>L)↑M12MIN,(L←YO12×A12)÷Y12MA
      X
[14]  ΔM12Y←ΔM12Y,⍎8↑(8×(M12MAX×Y12MIN)>L)⌽'M12MAX  L÷Y12MIN'
[15]  →LB3
[16] LB4:ΔM12Y←(¯1+2×(M12MIN×Y12MIN)≤L)↑M12MIN,(L←YO12×A12)÷Y1
      2MIN
[17]  ΔM12Y←ΔM12Y,⍎8↑(8×(M12MAX×Y12MAX)≤L)⌽'M12MAX  L÷Y12MAX'
[18] LB3:→(0=⍴⍴ΔM12←COMMON3 ΔM12X,ΔM12Y,M12MIN,M12MAX)↑0
[19]  →((X23MIN≤X23)∧X23MAX≥X23←(XO23×A23)÷MO23)/0
[20]  →((Y23MIN≤Y23)∧Y23MAX≥Y23←(YO23×A23)÷MO23)/0
[21]  →(0≥MUV←((KO23×A23*2)÷MO23)-+/(X23,Y23)*2)/0
[22]  →((K23MIN≤K23)∧K23MAX≥K23←MUV*.5)/0
[23]  ΔM43X←M43MIN,M43MAX
[24]  →(LB6,LB5,⎕LC[1]+1)[2+×XO43]
[25]  ΔM43X←(¯1+2×(M43MIN×X43MAX)>L)↑M43MIN,(L←XO43×A43)÷X43MA
      X
[26]  ΔM43X←ΔM43X,⍎8↑(8×(M43MAX×X43MIN)>L)⌽'M43MAX  L÷X43MIN'
[27]  →LB5
[28] LB6:ΔM43X←(¯1+2×(M43MIN×X43MIN)≤L)↑M43MIN,(L←XO43×A43)÷X4
      3MIN
[29]  ΔM43X←ΔM43X,⍎8↑(8×(M43MAX×X43MAX)≤L)⌽'M43MAX  L÷X43MAX'
[30] LB5:ΔM43Y←M43MIN,M43MAX
[31]  →(LB8,LB7,⎕LC[1]+1)[2+×YO43]
[32]  ΔM43Y←(¯1+2×(M43MIN×Y43MAX)>L)↑M43MIN,(L←YO43×A43)÷Y43MA
      X
[33]  ΔM43Y←ΔM43Y,⍎8↑(8×(M43MAX×Y43MIN)>L)⌽'M43MAX  L÷Y43MIN'
[34]  →LB7
[35] LB8:ΔM43Y←(¯1+2×(M43MIN×Y43MIN)≤L)↑M43MIN,(L←YO43×A43)÷Y4
      3MIN
[36]  ΔM43Y←ΔM43Y,⍎8↑(8×(M43MAX×Y43MAX)≤L)⌽'M43MAX  L÷Y43MAX'
[37] LB7:→(0=⍴⍴ΔM43←COMMON3 ΔM43X,ΔM43Y,M43MIN,M43MAX)↑0
```

```
[38]  K43MINS←K43MIN*2◇ K43MAXS←K43MAX*2
[40]  CK←KO43×A43*2
[41]  →(0≠CXY←((XO43*2)+(YO43*2))×A43*2)↑LB9
[42]  →(((M43MIN×K43MINS)>CK)∨CK>M43MAX×K43MAXS)↑0
[43]  ΔM43K←(¯1+2×(M43MIN×K43MAXS)>CK)↑M43MIN,CK÷K43MAXS
[44]  ΔM43K←ΔM43K,⍎10↑(10×(M43MAX×K43MINS)>CK)⌽'M43MAX    CK÷K
      43MINS'
[45]  →LB10
[46] LB9:→((K43MAXS<K2P),((K43MINS≤K2P)∧K2P≤K43MAXS),K43MINS>K
      2P←(CK*2)÷4×CXY)/(LB9+1),LB11,0
[47]  ΔM43K←(((CK-MUV),CK+MUV←((CK*2)-4×CXY×L)*.5)÷2×L,L←K43MI
      NS,K43MAXS)[1 2 4 3]
[48]  L←COMMON2 ΔM43K[3 4],ΔM43
[49]  →((0=ρρL)∧0=ρρMUV←COMMON2 ΔM43K[1 2],ΔM43)↑0
[50]  ΔM43←(((0=ρρL)↓0 0≠ρρL),(0=ρρMUV)↓0 0≠ρρMUV)/L,MUV
[51]  →(Z←1)/0
[52] LB11:ΔM43K←((CK-MUV),CK+MUV←((CK*2)-4×CXY×K43MINS)*.5)÷2×
      K43MINS
[53] LB10:→(0=ρρΔM43←COMMON2 ΔM43K,ΔM43)/0
[54]  Z←1
    ∇


    ∇ BEGIN GO;LO
[1]   ⍝ MAIN PROGRAM
[2]   →GO↑GO=1
[3]   GFSTUP
[4]   GFCLA
[5]   'PLEASE SELECT MECHANISM:'
[6]   '  1) FOUR-BAR'
[7]   '  2) SLIDER-CRANK'
[8]   '  3) WATT II'
[9]   '  4) STEPHENSON III'
[10]  →((0=ρρM_TY)∧∧/M_TY∊'1234')↑(¯1+'1234'⍳1↑M_TY←⍞)⊖LB1,LB2
      ,LB3,LB4
[11]  ⎕ARBOUT 7
[12]  →⎕LC[1]-2
[13] LB1:FBSPEC
[14]  'PLEASE WAIT...'
[15]  DASHES
[16] LB6:ERR←0
[17]  FBDVA 1 ¯1[⍎TYPE]
[18]  →LB5×⍳ERR=0
[19]  GFCLA
[20]  'AN ERROR OCCURRED DURING THE DISPLACEMENT ANALYSIS'
[21]  'OF THE 4-BAR. FROM THE LIST BELOW, SELECT ALL ITEMS'
[22]  'YOU WANT TO CHANGE (USE SPACES TO SEPARATE''EM) OR'
[23]  'ENTER 4 FOR QUITTING.'
[24]  ' '
[25]  ' 1) U1 V1 U4 V4 = ',(⍕U1),' ',(⍕V1),' ',(⍕U4),' ',⍕V4
[26]  ' 2) LENGTHS OF 12, 23, AND 43 = ',(⍕A12),' ',(⍕423),' '
      ,⍕A43
[27]  ' 3) TYPE OF DYAD 234 IS ',(2 ¯2[⍎TYPE])↑'+1-1'
```

```
[28]  ' 4) QUIT'
[29]  ⎕ARBOUT B1
[30]  FBTXT
[31]  ⎕ARBOUT 10⍴10
[32]  →(⎕LC[1]+4)×⍳∧/(MUV=4)∨(MUV←⎕)∊⍳3
[33]  ⎕ARBOUT 7
[34]  'TRY AGAIN'
[35]  →⎕LC[1]-3
[36]  →((⎕LC[1]+1),0)[1+∧/MUV∊4]
[37]  FBGEO MUV
[38]  →LB6
[39] LB5:AFLAG←12 2⍴'AT'
[40]  GFCLA
[41]  'ENTER:'
[42]  '   1) FOR GRAPHIC DISPLAY OF DATA FROM'
[43]  '      KIN. ANALYSIS;'
[44]  '   2) IF NO DISPLAY IS DESIRED;'
[45]  '   3) FOR QUITTING'
[46]  →((0=⍴⍴MUV)∧∧/MUV∊'123')↑(¯1+'123'⍳1↑MUV←⍞)⊖LB7,LB8,0
[47]  ⎕ARBOUT 7
[48]  'TRY AGAIN'
[49]  →⎕LC[1]-3
[50] LB7:FBKOUT
[51]  GFACT 1
[52]  GFMOV 0 770
[53]  ⎕ARBOUT 13
[54]  →LB5+2
[55] LB8:E12←E23←E43←2 1⍴0
[56]  F12←F23←F43←(2,N+1)⍴0
[57]  T12←T23←T43←(N+1)⍴0
[58]  LO←LOADS
[59]  FBSTATIC
[60]  →(LO=0)↑⎕LC[1]+3
[61]  CASE←'L'
[62]  STADYNOUT
[63]  M12←M23←M43←X12←X23←X43←Y12←Y23←Y43←K12←K23←K43←0
[64] LB9:MPIN
[65]  FBDYNAID
[66]  FBDYN
[67]  FBTOTAL
[68]  CASE←'T'
[69]  STADYNOUT
[70]  'WOULD YOU LIKE TO RETROCEDE AND'
[71]  'MODIFY THE CURRENT MASS PARAMETERS?'
[72]  ⍎'→',5↑(¯5+5×'YN'⍳1↑⍞)⌽'LB9  ⎕LC+3⎕LC+1'
[73]  ⎕ARBOUT 7
[74]  →⎕LC[1]-2
[75]  MPLIMITS
[76]  →⎕LC[1]+2
[77] LB11:MPLIMMOD
[78]  FBMPOLIM
[79]  'INDICATE ALTERNATIVE YOU WANT TO ADDRESS:'
[80]  '   1) SHAKING MOMENT SYNTHESIS'
[81]  '   2) SHAKING FORCE AND INPUT TORQUE SYNTHESIS'
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```
[82]  →⍎5↑(¯5+5×'21'⍳1↑SYNCASE←⍞)⌽'LB10 ⎕LC+3⎕LC+1'
[83]  ⎕ARBOUT 7
[84]  →⎕LC[1]-2
[85]  SMRP
[86] LB12:→LB11×⍳0=ITSMSYN
[87]  SMT←CURVE
[88] LB13:WFWIN
[89]  EXPIN
[90]  →(LB11,LB12,LB13,0)[⍎FBOPT]
[91] LB10:SMRP
[92] LB14:→LB11×⍳0=ITSMSYN
[93]  ITT←CURVE
[94]  →LB11×⍳0=SFSYN
[95]  SFT←CURVE
[96] LB15:WFWIN
[97]  EXPIN
[98]  →(LB11,LB14,LB15,0)[⍎FBOPT]
[99] LB2:
[100]LB3:
[101]LB4:
    ∇


      ∇ CALC L;⎕TX
 [1]     ⍝ FUNCTION CALLED BY 'CALCULATOR'. PERFORMS
 [2]     ⍝ OPERATION INDICATED BY L.
 [3]    ⎕TX←'ERR1←1◇→0'
 [4]    →L16×⍳L=16
 [5]    TT←T◇ ZZ←Z◇ YY←Y◇ XX←X◇ VVSTK←VSTK◇ →⍎'L',⍕L
 [11] L1:X←X*.5◇ →LC1
 [13] L2:X←1○X◇ →LC1
 [15] L3:X←2○X◇ →LC1
 [17] L4:X←X*2◇ →LC1
 [19] L5:X←¯1○X◇ →LC1
 [21] L6:X←¯2○X◇ →LC1
 [23] L7:X←*X◇ →LC1
 [25] L8:X←10*X◇ →LC1
 [27] L9:X←Y*X◇ →LC2
 [29] L10:X←⍟X◇ →LC1
 [31] L11:X←10⍟X◇ →LC1
 [33] L12:X←÷X◇ →LC1
 [35] L13:X←3○X◇ →LC1
 [37] L14:X←¯3○X◇ →LC1
 [39] LC1:VSTK[4;]←3↑('R',⍕R←R+1),' '◇ →0
 [41] L15:T←Z◇ Z←Y◇ Y←X
 [44]  VSTK←VSTK[2 3 4;],[1]VSTK[4;]◇ →0
 [46] L16:→ER×⍳(II=0)∧I=1
 [47]  MUV←X◇ X←XX◇ XX←MUV
 [50]  MUV←Y◇ Y←YY◇ YY←MUV
 [53]  MUV←Z◇ Z←ZZ◇ ZZ←MUV
 [56]  MUV←T◇ T←TT◇ TT←MUV
 [59]  MUV←VSTK◇ VSTK←VVSTK◇ VVSTK←MUV
 [62]  →(1+⎕LC[1])×(MUV←VVSTK[4;1])∊'NR'
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```
[63]  ⍎MUV,'←',MUV,'-1'◇ →0
[65] L17:T←Z◇ Z←Y◇ Y←X◇ X←IANG
[69]  VSTK←VSTK[2 3 4;],[1]'IA '◇ →0
[71] L18:GFMOV 0,H←H-10
[72]  ⎕ARBOUT 13 27 59
[73]  T←Z◇ Z←Y◇ Y←X◇ X←⍎⍞
[77]  VSTK←VSTK[2 3 4;],[1]3↑('N',⍕N←N+1),' '
[78]  ⎕ARBOUT 27 56◇ →0
[80] L19:X←Y÷X◇ →LC2
[82] L20:X←Y×X◇ →LC2
[84] L21:X←Y-X◇ →LC2
[86] L22:X←Y+X
[87] LC2:Y←Z◇ Z←T
[89]  VSTK←VSTK[1;],[1]VSTK[1 2;],[1]3↑('R',⍕R←R+1),' '◇ →0
[91] ER:ERR2←1◇ →0
    ∇
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    ∇ W←CALCULATOR INPANG;II;I;N;KIM;LV;ACC_IA;UNIT;RV;MUV;IANG;X;R;Y;Z;T;VSTK;H;CHPOS;COL;ROW;ERR1;ERR2
[1]   ⍝ CALCULATOR - CALCULATES VALUE OF A FUNCTION FOR EACH ELEMENT
[2]   ⍝              OF INPANG. LINKED 'SEGMENTS' OF DIFFERENT FUNCTIONS
[3]   ⍝              CAN ALSO BE CONSIDERED.
[4]   ⍝ INPANG     - VEC. OF INP. ANG. VALUES IN INTERVAL 0-2PI.
[5]   INPANG[⍴INPANG]←6.28
[6]   25 GFREG 150 200 850 700◇ 25 GFSCL 150 200 850 700
[8]   GFACT 25◇ TRC←'T'
[10]  KIM←(16 0 17,[1]5 3⍴18),19 20 0 21 0 22
[11]  KIM←((6 3⍴(⍳6),(3⍴0),7 8 9,(3⍴0),10 11 12),13 14 0,3⍴15),KIM
[12]  LV←¯1◇ W←ACC_IA←⍳0◇ UNIT←'U'
[15] LB3:GFCLA◇ ⎕ARBOUT 10 27 59
[17]  'YOU WANT TO CONSIDER THE INPUT ANGLE'
[18]  'FROM ',(⍕LV+LV<0),' TO WHAT VALUE?'
[19]  →(2+1↑⎕LC)×⍳~'U'∊UNIT
[20]  '(YOU''RE GOING TO BE ASKED ABOUT THE UNIT SUBSEQUENTLY)'
[21]  RV←⎕◇ ⎕ARBOUT 10◇ →LA1×⍳~'U'∊UNIT
[24]  'NOW SPECIFY THE UNIT BY PRESSING:'
[25]  '   R FOR RADIANS'
[26]  '   D FOR DEGREE'
[27] LA2:→LA1×⍳(UNIT←1↑⍞)∊'RD'
[28]  ⎕ARBOUT 7◇ →LA2
[30] LA1:→(3+1↑⎕LC)×⍳~(RV>(360,○2)[MUV←1+'R'∊UNIT])∨RV≤LV+LV<0
[31]  ⎕ARBOUT 7◇ →LB3
[33]  IANG←(((LV×MUV)<INPANG)∧INPANG≤RV×MUV←(((○1)÷180),1)[MUV])/INPANG
[34]  ⎕ARBOUT 10
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[35]  'IS THE FUNCTION CONSTANT FOR THIS INTERVAL OF THE INP.
      ANGLE?'
[36] LB4:→LB5×⍳1=(+/MUV∊'N')++/(MUV←⍞)∊'Y'
[37]  ⎕ARBOUT 7⋄ →LB4
[39] LB5:→LB6×⍳'N'∊MUV⋄ ⎕ARBOUT 10
[41]  'WHAT IS THAT CONSTANT?'⋄ X←(ρIANG)ρ⎕
[43] LB9:→LA3×⍳(ρX,1)≠ρIANG,1⋄ GFCLG
[45]  25 PLOT(ACC_IA,IANG),[.5]W,X⋄ GFACT 1
[47]  (250 100,[1.5]350 100)GFRECT 320 150,[1.5]710 150
[48]  ⎕ARBOUT 27 56
[49]  275 120 GFTXT 'OK',(5ρ' '),'UNDO LAST PART OF GRAPH'
[50]  ⎕ARBOUT 27 59
[51] LA4:→LB7×⍳∧/(250 100≤CHPOS),(CHPOS←2↑GFGCUR)≤320 150
[52]  →LB3×⍳∧/(350 100≤CHPOS),CHPOS≤710 150
[53]  ⎕ARBOUT 7⋄ →LA4
[55] LB7:W←W,X
[56]  ⍎(8×RV≥(360,6.28)[1+'R'∊UNIT])↑'GFCLA⋄→0'
[57]  LV←RV⋄ ACC_IA←ACC_IA,IANG⋄ →LB3
[60] LB6:II←R←N←0⋄ T←Z←Y←X←IANG⋄ VSTK←4 3ρ'IA '
[63]  H←770⋄ I←0⋄ GFCLG⋄ DRWCAL
[67]  (340 100,[1.5]590 100)GFRECT 530 150,[1.5]710 150
[68]  ⎕ARBOUT 27 56
[69]  365 120 GFTXT 'THAT''S IT',(9ρ' '),'RESET'
[70] LB2:GFMOV 200,701-110×II
[71]  ⎕ARBOUT 31 116 8 10 122 8 10 121 8 8 10 93 120
[72] LB1:MUV GFRECT(MUV←(2 4ρ(5ρ0),22 44 66)+(⍉4 2ρ(220+I×50),
      630-110×II))+⍉4 2ρ50 22
[73]  GFMOV 5 5+,MUV[;4]
[74]  ⎕ARBOUT 31,,(¯1+⎕AV⍳VSTK),4 4ρ8 8 8 10⋄ I←I+1
[76] LA5:→LB9×⍳∧/(340 100≤CHPOS),(CHPOS←2↑GFGCUR)≤530 150
[77]  →(-LV=¯1)+LB3×⍳∧/(590 100≤CHPOS),CHPOS≤710 150
[78]  COL←220 300 380 460 540 630 640 830≤CHPOS[1]
[79]  →LA3×⍳~1∊COL←COL∧290 370 450 530 630 640 820 900≥CHPOS[1
      ]
[80]  ROW←430 370 320 310 260 250≥CHPOS[2]
[81]  →LA3×⍳~1∊ROW←ROW∧380 320 310 260 250 200≤CHPOS[2]
[82]  ERR1←ERR2←0⋄ CALC KIM[(ROW⍳1);COL⍳1]
[84]  →LB8×⍳ERR1=0⋄ ⎕ARBOUT 7⋄ GFMOV 0,H←H-10
[87]  ⎕ARBOUT 13 27 59⋄ ⎕EM[1;]⋄ ⎕ARBOUT 27 56
[90]  →LA5⋄LA3:⎕ARBOUT 7⋄ →LA5
[93] LB8:→LA3×⍳ERR2=1⋄ II←II+I=15
[95]  I←(I,0)[1+I=15]⋄ →(LB1,LB2)[1+I=0]
    ∇


    ∇ Z←CALCOF1;MUV;ACTL
[1]    ⍝ CALCULATES OBJ.FNC. FOR CASE INVOLVING
[2]    ⍝ S.MOMENT SYNTHESIS
[3]    ACTL←⍳Z←0
[4]    FBRI⋄ F1T←F1L+F1I⋄ F2T←F2L+F2I⋄ F3T←F3L+F3I⋄ F4T←F4L+F4I

[9]    →(⎕LC[1]+2)×⍳WF[1]=0
[10]   →(WORST[1]<¯1↑ACTL←ACTL,⌈/(+⌿F1T*2)*.5)/0
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```
[11]  →(⎕LC[1]+2)×⍳WF[2]=0
[12]  →(WORST[2]<¯1↑ACTL←ACTL,⌈/(+/F2T*2)*.5)/0
[13]  →(⎕LC[1]+2)×⍳WF[3]=0
[14]  →(WORST[3]<¯1↑ACTL←ACTL,⌈/(+/F3T*2)*.5)/0
[15]  →(⎕LC[1]+2)×⍳WF[4]=0
[16]  →(WORST[4]<¯1↑ACTL←ACTL,⌈/(+/F4T*2)*.5)/0
[17]  FBFTI⋄ SFT←SFL+SFI
[19]  →(⎕LC[1]+2)×⍳WF[5]=0
[20]  →(WORST[5]<¯1↑ACTL←ACTL,⌈/(+/SFT*2)*.5)/0
[21]  ITT←ITL+ITI
[22]  →(⎕LC[1]+2)×⍳WF[6]=0
[23]  →(WORST[6]<¯1↑ACTL←ACTL,(⌈/ITT)-⌊/ITT)/0
[24]  →(⎕LC[1]+2)×⍳WF[7]=0
[25]  →(WORST[7]<¯1↑ACTL←ACTL,((+/ITT*2)÷⍴ITT)*.5)/0
[26]  →(⎕LC[1]+2)×⍳WF[8]=0
[27]  →(WORST[8]<¯1↑ACTL←ACTL,⌈/|ITT)/0
[28]  Z←1⋄ MUV←WF≠0
[30]  OF←+/(MUV/WF)×(1-ACTL÷MUV/WORST)*MUV/EXP
    ∇


    ∇ Z←CALCOF2;MUV;ACTL
[1]   ⍝ CALCULATES OBJ.FNC. FOR CASE INVOLVING
[2]   ⍝ I.TORQUE AND S.FORCE SYNTHESES.
[3]   ⍝ CALCULATES OBJ. FUNCTION FOR⋄ ⍝ CASE OF S.FORCE+I.TORQ
      UE SYNTHESIS
[5]   ⍝ OF 4-BAR LINKAGE⋄ ACTL←⍳Z←0⋄ FBRI⋄ F1T←F1L+F1I⋄ F2T←F2
      L+F2I
[10]  F3T←F3L+F3I⋄ F4T←F4L+F4I
[12]  →(⎕LC[1]+2)×⍳WF[1]=0
[13]  →(WORST[1]<¯1↑ACTL←ACTL,⌈/(+/F1T*2)*.5)/0
[14]  →(⎕LC[1]+2)×⍳WF[2]=0
[15]  →(WORST[2]<¯1↑ACTL←ACTL,⌈/(+/F2T*2)*.5)/0
[16]  →(⎕LC[1]+2)×⍳WF[3]=0
[17]  →(WORST[3]<¯1↑ACTL←ACTL,⌈/(+/F3T*2)*.5)/0
[18]  →(⎕LC[1]+2)×⍳WF[4]=0
[19]  →(WORST[4]<¯1↑ACTL←ACTL,⌈/(+/F4T*2)*.5)/0
[20]  FBFTI⋄ FBSMI
[22]  SMT←SML+SMI
[23]  →(⎕LC[1]+2)×⍳WF[5]=0
[24]  →(WORST[5]<¯1↑ACTL←ACTL,(⌈/SMT)-⌊/SMT)/0
[25]  →(⎕LC[1]+2)×⍳WF[6]=0
[26]  →(WORST[6]<¯1↑ACTL←ACTL,((+/SMT*2)÷⍴SMT)*.5)/0
[27]  →(⎕LC[1]+2)×⍳WF[7]=0
[28]  →(WORST[7]<¯1↑ACTL←ACTL,⌈/|SMT)/0⋄ Z←1
[30]  MUV←WF≠0
[31]  OF←+/(MUV/WF)×(1-ACTL÷MUV/WORST)*MUV/EXP
    ∇
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```
    ∇ Z←COMMON2 INT
[1]    ⍝ FINDS INTERSECTION OF INTERVALS INT[1 2]
[2]    ⍝ AND INT[3 4].
[3]    ⍝ Z=0 IF THERE IS NO INTERSECTION.
[4]    ⍝ IF INTERSECTION IS A POINT, Z[1]=Z[2].
[5]    ⍝ INT[1]=INT[2] OR INT[3]=INT[4] IS OK.
[6]   →((</INT[4 1])∨</INT[2 3])↑Z←0
[7]   Z←INT[(⍋INT)[2 3]]
    ∇


    ∇ Z←COMMON3 INT;MUV
[1]    ⍝ FINDS INTERSECTION OF INTERVALS INT[1 2],
[2]    ⍝ INT[3 4] AND INT[5 6].
[3]    ⍝ SEE COMMENTS IN COMMON2
[4]   →(0=⍴⍴MUV←COMMON2 4↑INT)↑Z←0
[5]   →(0=⍴⍴MUV←COMMON2 MUV,¯2↑INT)↑0
[6]   Z←MUV
    ∇


    ∇ DASHES;ABC;MUV
[1]   ⍝ CREATES BUFFERS FOR DASHED LINES
[2]   ⍝ USED BY PLOT AND PLOT3.
[3]   TRC←'A'
[4]   GVRDT←1 2 3 4,4 12⍴1 ¯12↑GVRDT
[5]   GFACT 2◇ ABC←25◇ DASHTYPE←3
[8]  L1:(620,MUV)GFDALINE 980,MUV←750-ABC
[9]   →L1×⍳300≠ABC←ABC+25
[10]  GFACT 3◇ ABC←25
[12] L2:(MUV,400)GFDALINE(MUV←270+ABC),40
[13]  →L2×⍳300≠ABC←ABC+25
[14]  GFACT 4◇ ABC←44
[16] L3:(MUV,585)GFDALINE(MUV←450+ABC),195
[17]  →L3×⍳528≠ABC←ABC+44
[18]  B2←GFRBUF 2
[19]  B3←GFRBUF 3
[20]  B4←GFRBUF 4
[21]  GFPBUF 2 3 4◇ TRC←'AT'
    ∇


    ∇ DRWCAL;A;B;H;W1;W2;W3;Δ;HD;LL;UR;VD;MUV1;MUV2
[1]    ⍝ DRAWS A CALCULATOR IN THE ACTIVE REGION
[2]    ⍝ TO BE USED BY 'CALCULATOR'.
[3]   A←220◇ B←200◇ H←50◇ W1←70
[7]   W2←90◇ W3←2×W2◇ Δ←10
[10]  HD←W1+Δ
[11]  LL←(⍉4 2⍴A,B)+VD←2 4⍴(5⍴0),(H+Δ)×⍳3
[12]  UR←(⍉4 2⍴(A+W1),B+H)+VD
[13]  LL←LL,(⍉4 2⍴(A+HD),B)+VD
[14]  UR←UR,(⍉4 2⍴(A+HD+W1),B+H)+VD
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[15]  LL←LL,(⍉4 2⍴(A+2×HD),B)+VD
[16]  UR←UR,(⍉4 2⍴(A+W1+2×HD),B+H)+VD
[17]  LL←LL,(MUV1←A+3×HD),B
[18]  UR←UR,(A+W1+3×HD),B+Δ+2×H
[19]  LL←LL,(⍉2 2⍴MUV1←MUV1,B+2×H+Δ)+MUV2←2 2⍴(3⍴0),H+Δ
[20]  UR←UR,(⍉2 2⍴MUV1←MUV1+W1,H)+MUV2
[21]  LL←LL,MUV1←MUV1+Δ◇ UR←UR,MUV1+W2,H
[23]  LL←LL,MUV1←MUV1+(W2+Δ),0◇ UR←UR,MUV1←MUV1+W3,H
[25]  LL←LL,MUV2←(A+4×HD),B
[26]  UR←UR,MUV2←MUV2+(W2+Δ+W3),H+2×H+Δ
[27]  LL←LL,(⍉4 2⍴MUV1←(MUV1[1]+Δ),B)+VD
[28]  UR←UR,(⍉4 2⍴MUV1+W1,H)+VD
[29]  LL←LL,MUV1←(A,B)-2×Δ
[30]  UR←UR,MUV2←MUV2+(W1,H)+3×Δ
[31]  LL←LL,MUV1-1◇ UR←UR,MUV2+1
[33]  LL GFRECT UR◇ ⎕ARBOUT 27 56
[35]  (25 20+A,B)GFTXT 'LN',(3⍴' '),'LOG',(3⍴' '),'1/X',(14⍴'
      '),'KEYBOARD',(7⍴' '),'Y+X'
[36]  (MUV1←20 10+A,B)GFTXT(18⍴' '),'R'
[37]  (MUV1←0 20+MUV1)GFTXT(18⍴' '),'E'
[38]  (MUV1←0 20+MUV1)GFTXT(18⍴' '),'T'
[39]  (MUV1←0 20+MUV1)GFTXT(18⍴' '),'N'
[40]  (MUV1←0 20+MUV1)GFTXT(18⍴' '),'E'
[41]  (MUV1←25 18+A,B+VD←H+Δ)GFTXT(5⍴' '),'10',(4⍴' '),'Y',(11
      ⍴' '),'NUMBER  THRU THE',(4⍴' '),'Y-X'
[42]  0 1 1 1 1 1 1 1 1 1 GFVABS(⍉10 2⍴MUV1)+2 10⍴10 6 0 ¯4 ¯4
       0 6 10 10 ¯4 4 0 0 4 12 16 16 12 9 9
[43]  (MUV2←2 14+MUV1)GFTXT(1⍴' '),'X',(5⍴' '),'X',(4⍴' '),'X'

[44]  (MUV1←24 14+A,B+2×VD)GFTXT 'X',(4⍴' '),'SIN',(2⍴' '),'CO
      S',(3⍴' '),'TAN',(3⍴' '),'CLICK HERE TO INPUT',(2⍴' '),'
      Y×X'
[45]  (0 15+MUV1)GFTXT ' 2',(5⍴' '),'-1',(3⍴' '),'-1',(4⍴' '),
      '-1'
[46]  (MUV2←MUV1+6,VD)GFTXT 'X',(4⍴' '),'SIN',(2⍴' '),'COS',(3
      ⍴' '),'TAN',(3⍴' '),'UNDO'
[47]  (MUV1+0,VD)GFTXT(43⍴' '),'Y÷X'
[48]  (MUV1←MUV2-8 5)GFTXT(29⍴' '),'(IN RADIANS)'
[49]  (MUV1+0 20)GFTXT(29⍴' '),'IA-INP.ANGLE'
[50]  0 1 1 1 GFVABS(⍉4 2⍴MUV1)+2 4⍴¯15 ¯10 ¯1 25 20 0 30 30
[51]  ⎕ARBOUT 27 59
    ∇


    ∇ DRWFB;C21;C32;C43;T11;T12;MUV;T21;T22
[1]    ⍝ DRAWS A FOUR-BAR IN THE ACTIVE REGION.
[2]   (C1←750 100)GFCIRC 8 40
[3]   (C2←800 190)GFCIRC 8 40
[4]   (C3←960 230)GFCIRC 8 40
[5]   (C4←970 100)GFCIRC 8 40
[6]   C21←8×C21÷(+/(C21←C2-C1)*2)*.5
[7]   (C1+C21)GFLINE C2-C21
[8]   C32←8×C32÷(+/(C32←C3-C2)*2)*.5
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[9]   (C2+C32)GFLINE C3-C32
[10]  C43←8×C43÷(+/(C43←C4-C3)*2)*.5
[11]  (C3+C43)GFLINE C4-C43
[12]  T11←⌊.5+C1+8×T11÷(+/(T11←735 80-C1)*2)*.5
[13]  T12←⌊.5+C1+8×T12÷(+/(T12←765 80-C1)*2)*.5
[14]  0 1 1 1 GFVABS⍉4 2ρT11,735 80 765 80,T12
[15]  725 80 GFLINE 775 80
[16]  MUV←725 70 730 80 740 70 745 80 755 70 760 80 770 70 775
       80
[17]  0 1 0 1 0 1 0 1 GFVABS⍉8 2ρMUV
[18]  T21←⌊.5+C4+8×T21÷(+/(T21←955 80-C4)*2)*.5
[19]  T22←⌊.5+C4+8×T22÷(+/(T22←985 80-C4)*2)*.5
[20]  0 1 1 1 GFVABS⍉4 2ρT21,955 80 985 80,T22
[21]  945 80 GFLINE 995 80
[22]  MUV←945 70 950 80 960 70 965 80 975 70 980 80 990 70 995
       80
[23]  0 1 0 1 0 1 0 1 GFVABS⍉8 2ρMUV
[24]  750 100 GFARC 40 ¯25 75 60
[25]  1 0 1 GFVINC⍉3 2ρ2 ¯10 ¯2 10 9 ¯3
[26]  0 1 1 GFVABS⍉3 2ρ700 140 700 50 785 50
[27]  MUV←696 132 700 140 704 132 777 54 785 50 777 46
[28]  0 1 1 0 1 1 GFVABS⍉6 2ρMUV
[29]  MUV←720 40 725 50 735 40 740 50 750 40 755 50 765 40 770
       50
[30]  0 1 0 1 0 1 0 1 GFVABS⍉8 2ρMUV
[31]  650 0 GFRECT 1020 280
    ∇


    ∇ DRWFB2
[1]   ⍝ DRAWS ON SCREEN THE 4-BAR
[2]   ⍝ STORED IN BUFFER B1.
[3]   ⎕4RBOUT B1
[4]   GFACT 1◇ FBAXES◇ FBTXT
[7]   GFMOV 0 779◇ ⎕ARBOUT 13
    ∇


    ∇ DRWSTAR X;STAR
[1]   ⍝ DRAWS STARS FOR PRECISION POINTS(SYNTHESIS)
[2]   →(0=ρX)/0
[3]   →(1=ρρX)/⎕LC[1]+2◇ X←,⍉X
[5]   STAR←8 2ρ0 ¯3 0 3 ¯3 0 3 0 ¯3 ¯3 3 3 3 ¯3 ¯3 3
[6]   (8ρ0 1)GFVABS⍉(8 2ρ2↑X)+STAR
[7]   →(⎕LC[1]-1)×0≠ρX←2↓X
    ∇
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    ∇ EXPIN;KK;NJ;I
[1]    ⍝ ALLOWS USER TO SPECIFY EXPONENT OF SCORE CURVE
[2]    GFCLA
[3]    'THE SCORE SIGNIFYING PERFORMANCE OF THE LINKAGE RELATIV
       E TO'
[4]    'A DESIRABLE CONDITION (FOR EX: MAX MAG OF 4 BEARING REA
       CTION)'
[5]    'WILL BE ASSIGNED THROUGH THE FORMULA:'
[6]    ⍳0⋄ 60⍴'~'⋄ ⍳0
[9]    '    SCORE = (1-(ACTUAL VALUE÷WORST VALUE))*EXP'
[10]   ⍳0⋄ 60⍴'~'
[12]   'WHERE ''*'' MEANS EXPONENTIATION.'
[13]   'NEXT  YOU''LL BE PROVIDED WITH A MEANS FOR CHOOSING AN'

[14]   'ADEQUATE VALUE OF ''EXP'' FOR  EACH  DESIRABLE  CONDITI
       ON.'
[15]   ⍳0⋄ 'PRESS ''RETURN'' TO CONTINUE.'⋄ KK←⍞
[18]   EXP←⍳0⋄ NJ←1↑⍴JOINTS⋄ I←1
[21]   EXP←EXP,SCOREPLOT 3 10⍴'MAX MAG OFFORCE AT  JOINT ',(⍕I)
       ,'   '
[22]   →(⎕LC[1]-1)×⍳NJ≥I←I+1
[23]   →LB1×⍳SYNCASE='2'
[24]   EXP←EXP,SCOREPLOT 3 10⍴'MAX MAG OFSHAKING   FORCE      '
[25]   EXP←EXP,SCOREPLOT 3 9⍴'PTP VALUEOF INPUT TORQUE   '
[26]   EXP←EXP,SCOREPLOT 3 9⍴'RMS VALUEOF INPUT TORQUE   '
[27]   EXP←EXP,SCOREPLOT 3 10⍴'MAX ABS   VALUE OF  INP TORQUE'
[28]   →0
[29] LB1:EXP←EXP,SCOREPLOT 3 10⍴'PTP VALUE OF SHAKINGMOMENT
       '
[30]   EXP←EXP,SCOREPLOT 3 10⍴'RMS VALUE OF SHAKINGMOMENT    '
[31]   EXP←EXP,SCOREPLOT 3 8⍴'MAX ABS VALUE OFS.MOMENT'
    ∇
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```
    ∇ FBAXES;A;I;FLAG;MUV;MUV2;XAXIS;YAXIS;JOINT;S
[1]    ⍝ DRAWS LOCAL AXES OF 4-BAR
[2]    TRC←'T'⋄ A←3 2⍴(C2-C1),(C3-C2),C3-C4
[4]    I←1⋄ LINKS←3 2⍴'122343'
[6]   L1:MUV2←30×MUV←MUV÷(+/(MUV←,A[I;])*2)*.5
[7]    XAXIS←MUV2,⌽(150 ROTVEC S),(-MUV),[1.1]MUV←¯150 ROTVEC S
       ←6×MUV
[8]    YAXIS←90 ROTVEC XAXIS
[9]    GFMOV JOINT←⍎'C',LINKS[I;FLAG←1+1=⍎'FLAG',LINKS[I;]]
[10]   1 1 0 1 GFVINC⌊((⍴XAXIS)⍴.5)+XAXIS×1 ¯1[FLAG]
[11]   GFMOV JOINT
[12]   1 1 0 1 GFVINC⌊((⍴YAXIS)⍴.5)+YAXIS×1 ¯1[FLAG]
[13]   →L1×⍳4>I←I+1
[14]   LINKS←LINKS,(FLAG12,FLAG23,FLAG43)⌽LINKS
[15]   TBL←((FLAG12×180,A12),[1](FLAG23×180,A23),[.5]FLAG43×180
       ,A43),3⍴0
[16]   GFMOV 0 500⋄ ⎕ARBOUT 13
    ∇
```

```
    ∇ FBDVA TYPE;DEN;R24;D;COSΔ;Δ;R43;EL;G23;G43;H23;H43;⎕TX
[1]    ⍝ DISP./VEL./ACC. ANALYSIS OF 4-BAR
[2]    ⎕TX←'ERR←1◇→0'
[3]    EL←ρPHI1̲2←(0,ιN)×(○2)÷N
[4]    U2←U1+,¯1 0↓R12←A12×(2○PHI12),[.5]1○PHI12
[5]    R24←((ELρU4),[.5]ELρV4)-R12+⍉(EL,2)ρU1,V1
[6]    COSΔ←(D+(-A43*2)+A23*2)÷2×A23×(D←+/R24*2)*.5
[7]    Δ←COSΔ ARG TYPE×(1-COSΔ*2)*.5
[8]    PHI23←PHI23-(○2)×(○2)≤PHI23←Δ+R24[1;]ARG R24[2;]
[9]    R43←(A23×(2○PHI23),[.5]1○PHI23)-R24
[10]   PHI43←R43[1;]ARG R43[2;]◇ V2←V1+R12[2;]
[12]   U3←U1+,¯1 0↓V3←R12+R24+R43◇ V3←V1+,1 0↓V3
[14]   U2D←-A12×WI×1○PHI12◇ V2D←A12×WI×2○PHI12
[16]   U2DD←-A12×(WI*2)×2○PHI12◇ V2DD←-A12×(WI*2)×1○PHI12
[18]   PHI23D←WI×G23←(A12×1○PHI12-PHI43)÷A23×DEN←1○PHI43-PHI23
[19]   PHI43D←WI×G43←(A12×1○PHI12-PHI23)÷A43×DEN
[20]   U3D←-A43×PHI43D×1○PHI43◇ V3D←A43×PHI43D×2○PHI43
[22]   H23←((-A43×G43*2)+(A12×2○PHI43-PHI12)+A23×(G23*2)×2○PHI4
       3-PHI23)÷A23×DEN
[23]   H43←((A23×G23*2)+(A12×2○PHI23-PHI12)-A43×(G43*2)×2○PHI23
       -PHI43)÷A43×DEN
[24]   PHI23DD←H23×WI*2◇ PHI43DD←H43×WI*2
[26]   U3DD←-A43×(PHI43DD×1○PHI43)+(PHI43D*2)×2○PHI43
[27]   V3DD←A43×(PHI43DD×2○PHI43)-(PHI43D*2)×1○PHI43
    ∇


    ∇ FBDYN;MXY23G;VEC
[1]    ⍝ DYNAMIC ANAL. OF 4-BAR FOR GIVEN MASS PARAMETERS
[2]    MXY23G←PHI23 ROTVEC3⍉((N+1),2)ρ(XO23,YO23)×A23
[3]    VEC←((-/MXY23G×V2DD,[.5]U2DD)+PHI23DD×KO23×A23*2),[.5]PH
       I43DD×KO43×A43*2
[4]    F3I←((VEC[1;]×C22)-VEC[2;]×C12)÷DETC
[5]    F3I←F3I,[.5]((C11×VEC[2;])-C21×VEC[1;])÷DETC
[6]    F4I←F3I+((XO43×INA43)-YO43×DANI43),[.5](XO43×DANI43)+YO4
       3×INA43
[7]    F2I←F3I-(M23×U2DD,[.5]V2DD)+((XO23×INA23)-YO23×DANI23),[
       .5](XO23×DANI23)+YO23×INA23
[8]    F1I←(((XO12×INA12)-YO12×DANI12),[.5](XO12×DANI12)+YO12×I
       NA12)-F2I
[9]    ITI←A12×-/((1○PHI12),[.5]2○PHI12)×F2I◇ SFI←-F1I+F4I
    ∇


    ∇ FBFTI̲
[1]    ⍝ CALCULATES S.FORCE+INP.TOR. DUE TO OPTIMUM M.P.
[2]    ITI̲←A12×-/((1○PHI12),[.5]2○PHI12)×F2I̲
[3]    SFI̲←-F1I̲+F4I̲
    ∇
```

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     ∇ FBGEO Z;MUV
[1]    ⍝ ASKS FOR 4-BAR GEOMETRY
[2]    →LB1×⍳~1∊Z
[3]    'ENTER COORDINATES OF PIVOTS 1 AND 4 (U1 V1 U4 V4):'
[4]    →(⎕LC[1]+4)×⍳5=⍴1,MUV←⎕
[5]    ⎕ARBOUT 7⋄ 'TRY AGAIN'⋄ →⎕LC[1]-3
[8]    U1←MUV[1]⋄ V1←MUV[2]⋄ U4←MUV[3]⋄ V4←MUV[4]
[12]  LB1:→LB2×⍳~2∊Z
[13]   'ENTER LENGTHS OF CRANK, COUPLER, AND LINK 43'
[14]   '(L12 L23 L43):'
[15]   →(⎕LC[1]+4)×⍳4=⍴1,MUV←⎕
[16]   ⎕ARBOUT 7⋄ 'TRY AGAIN'⋄ →⎕LC[1]-3
[19]   A12←MUV[1]⋄ A23←MUV[2]⋄ A43←MUV[3]
[22]  LB2:→(~3∊Z)↑0
[23]   MUV←0
[24]  L1:'WHAT IS THE TYPE OF DYAD 234?'
[25]   '  1) +1'
[26]   '  2) -1'
[27]   (9×MUV)↓'  3) HELP'
[28]   →(⎕LC[1]+3)×⍳(0=⍴⍴TYPE)∧∧/(TYPE←⍞)∊(-MUV)↓'123'
[29]   ⎕ARBOUT 7⋄ →⎕LC[1]-2⋄ →(~MUV←TYPE='3')↑0
[32]   'THE TYPE IS +1 IF WHEN YOU STAND AT JOINT 2 AND'
[33]   'LOOK AT JOINT 4, JOINT 3 IS AT YOUR LEFT.'
[34]   'OTHERWISE THE TYPE IS -1 .'
[35]   ' '⋄ →L1⋄L2:SPEC
     ∇


     ∇ FBRI̲;MXY23G;VEC
[1]    ⍝ CALCULATES REACTIONS DUE TO M.P. FROM OPTIMIZATION
[2]    MXY23G←PHI23 ROTVEC3⍉((N+1),2)⍴(XO23,YO23)×A23
[3]    VEC←((-⌿MXY23G×V2DD,[.5]U2DD)+PHI23DD×KO23×A23*2),[.5]PH
       I43DD×KO43×A43*2
[4]    F3I̲←((VEC[1;]×C22)-VEC[2;]×C12)÷DETC
[5]    F3I̲←F3I,[.5]((C11×VEC[2;])-C21×VEC[1;])÷DETC
[6]    F4I̲←F3I̲+((XO43×INA43)-YO43×DANI43),[.5](XO43×DANI43)+YO4
       3×INA43
[7]    F2I̲←F3I̲-(MO23×U2DD,[.5]V2DD)+((XO23×INA23)-YO23×DANI23),
       [.5](XO23×DANI23)+YO23×INA23
[8]    F1I̲←(((XO12×INA12)-YO12×DANI12),[.5](XO12×DANI12)+YO12×I
       NA12)-F2I̲
     ∇


     ∇ FBSMI
[1]    ⍝ CALC. S.MOMENT DUE TO GIVEN M.P.'S
[2]    SMI←-((⌽⍉F4I)-.×(U4,V4)-RPOINT)+((⌽⍉F1I)-.×(U1,V1)-RPOIN
       T)+ITI
     ∇
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      ∇ FBSML
 [1]    ⍝ CALC. S.MOMENT DUE TO EXT. LOADS
 [2]    SML←-((⌽⍉FL_43)-.×(U4,V4)-RPOINT)+((⌽⍉FL_12)-.×(U1,V1)-R
        POINT)+ITL
      ∇


      ∇ FBSMI̲
 [1]    ⍝ CALC. S.MOMENT DUE TO M.P. FROM OPTIMIZATION
 [2]    SMI̲←-((⌽⍉F4I̲)-.×(U4,V4)-RPOINT)+((⌽⍉F1I̲)-.×(U1,V1)-RPOIN
        T)+ITI̲
      ∇


      ∇ FBSPEC
 [1]    ⍝ USED FOR SPEC. OF 4-BAR
 [2]    GFCLG⋄ GFACT 1⋄ DRWFB
 [5]    B1←GFRBUF 1⋄ GFPBUF 1⋄ FBTXT
 [8]    FBGEO 1 2 3⋄ SPEC
 [10]   FLAG12←FLAG23←FLAG43←0
 [11] LB1:GFCLG⋄ ⎕4RBOUT B̲1⋄ FBTXT
 [14]   FBAXES⋄ T̲R̲C̲←'AT'⋄ ⎕ARBOUT 13
 [17]   'ARE THE LOCAL COORD. SYSTEMS SHOWN OK?'
 [18]   ⍎'→',5↑(¯5+5×'NY'⍳1↑⍞)⌽'LB2  00000⎕LC+1'
 [19]   ⎕ARBOUT 7⋄ →⎕LC[1]-2
 [21] LB2:'WHAT COORD. SYSTEMS MUST BE CHANGED?'
 [22]   '   1) LINK 12'
 [23]   '   2) LINK 23'
 [24]   '   3) LINK 43'
 [25]   'FOR EXAMPLE, YOU MAY ENTER: 2 3'
 [26]   →LB3×⍳(∧/MUV∊1 2 3)∧3≥⍴,MUV←⎕
 [27]   ⎕ARBOUT 7⋄ 'TRY AGAIN'⋄ →⎕LC[1]-3
 [30] LB3:FLAG12←⍎((1∊MUV)↑'~'),'FLAG12'
 [31]   FLAG23←⍎((2∊MUV)↑'~'),'FLAG23'
 [32]   FLAG43←⍎((3∊MUV)↑'~'),'FLAG43'⋄ →LB1
      ∇


      ∇ FBSTATIC;E12G;E23G;E43G;VEC
 [1]    ⍝ 4-BAR STATIC ANALYSIS
 [2]    E12G←(PHI12×180÷○1)ROTVEC2⍎14↑(14×1=¯1↑⍴E12)⌽('E12',11⍴'
         '),'⍉((N+1),2)⍴E12'
 [3]    E23G←(PHI23×180÷○1)ROTVEC2⍎14↑(14×1=¯1↑⍴E23)⌽('E23',11⍴'
         '),'⍉((N+1),2)⍴E23'
 [4]    E43G←(PHI43×180÷○1)ROTVEC2⍎14↑(14×1=¯1↑⍴E43)⌽('E43',11⍴'
         '),'⍉((N+1),2)⍴E43'
 [5]    VEC←((-⌿E23G[2 1;]×F23)-T23),[.5](-⌿E43G[2 1;]×F43)-T43
 [6]    DETC←((C11←-A23×1○PHI23)×C22←-A43×2○PHI43)-(C12←A23×2○PH
        I23)×C21←A43×1○PHI43
 [7]    FL23_←((VEC[1;]×C22)-VEC[2;]×C12)÷DETC
 [8]    F3L←FL23_←FL23_,[.5]((C11×VEC[2;])-C21×VEC[1;])÷DETC
 [9]    F4L←FL_43←FL23_-F43
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[10]  F2L←FL12_←FL23_+F23
[11]  F1L←FL_12←-FL12_+F12
[12]  ITL←(412×(-FL12_[2;]×2○PHI12)+(FL12_[1;]×1○PHI12))+(-/E1
      2G[2 1;]×F12)-T12
[13]  SFL←-FL_12+FL_43
[14]  JOINTS←LINKS[1;3 4],[1]LINKS[;3 4]
    ∇


    ∇ FBTOTAL
[1]   ⍝ CALC. TOTAL REACTIONS, S.FORCE AND I.TOR
[2]   F1T←F1L+F1I⋄ F2T←F2L+F2I⋄ F3T←F3L+F3I
[5]   F4T←F4L+F4I⋄ SFT←SFL+SFI⋄ ITT←ITL+ITI
    ∇


    ∇ FBTXT;C21;C32;C43;T11;T12;MUV;T21;T22
[1]   ⍝ CREATES TEXT IN 4-BAR PICTURE
[2]   740 112 GFTXT '1'
[3]   780 195 GFTXT '2'
[4]   945 240 GFTXT '3'
[5]   985 110 GFTXT '4'
[6]   795 134 GFTXT '.'
[7]   795 120 GFTXT 'Φ  =CONST.'
[8]   803 115 GFTXT '12'
[9]   785 30 GFTXT 'U'
[10]  690 38 GFTXT 'O'
[11]  688 135 GFTXT 'V'
[12]  GFMOV 0 779⋄ ⎕ARBOUT 13
    ∇


    ∇ FBDYNAID;MUV;S23;C23;S43;C43
[1]   ⍝ GENERATES VARS. FOR FBDYN
[2]   INA12←(MUV←-A12×WI*2)×2○PHI12
[3]   DANI12←MUV×1○PHI12
[4]   INA23←-A23×(PHI23DD×S23←1○PHI23)+(MUV←PHI23D*2)×C23←2○PH
      I23
[5]   DANI23←A23×(PHI23DD×C23)-S23×MUV
[6]   INA43←-A43×(PHI43DD×S43←1○PHI43)+(MUV←PHI43D*2)×C43←2○PH
      I43
[7]   DANI43←A43×(PHI43DD×C43)-S43×MUV
[8]   XO12←M12×X12÷A12⋄ YO12←M12×Y12÷A12
[10]  XO23←M23×X23÷A23⋄ YO23←M23×Y23÷A23
[12]  KO23←M23×((X23*2)+(Y23*2)+K23*2)÷A23*2
[13]  XO43←M43×X43÷A43⋄ YO43←M43×Y43÷A43
[15]  KO43←M43×((X43*2)+(Y43*2)+K43*2)÷A43*2
    ∇
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    ∇ Z←FBITCOEF;D223;D323;D443;CO;IN
[1]   ⍝ CALC. COEFS. FOR EQUALITY CONSTRAINTS
[2]   ⍝ FROM I.TORQUE SYNTHESIS
[3]   PRCPTSL←PHI12[IN←1+⌊.5+PRCPTSL[1;]÷360÷N],[.5]PRCPTSL[2;
      ]
[4]   PRCPTSLD←(PRCPTSL[1;]×180÷○1),[.5]PRCPTSL[2;]
[5]   D223←(((U2D×U3DD)+V2D×V3DD)+(U3D×U2DD)+V3D×V2DD)÷WI
[6]   D323←(((U3D×V2DD)-V3D×U2DD)-(U2D×V3DD)-V2D×U3DD)÷WI
[7]   D443←((U3D×U3DD)+V3D×V3DD)÷WI
[8]   CO←(PRCPTSL[2;]-ITL[IN])⌹D223[IN],D323[IN],[1.5]D443[IN]

[9]    ⍝NOW NEC.CONDITIONS ARE GOING TO BE CHECKED
[10]  Z←0
[11]  →(((XO23MIN-KO23MAX)≤CO[1])⍲CO[1]≤XO23MAX-KO23MIN)/0
[12]  →((YO23MIN≤CO[2])⍲CO[2]≤YO23MAX)/0
[13]  →(((KO23MIN+KO43MIN)≤CO[3])⍲CO[3]≤KO23MAX+KO43MAX)/0
[14]  KITCO←ITCO⋄ ITCO←CO
[16]  KCURVE←CURVE
[17]  CURVE←ITT←ITL+ITI←(ITCO[1]×D223)+(ITCO[2]×D323)+ITCO[3]×
      D443
[18]  KPRCPTS←PRCPTSLD⋄ KFIRST←FIRST⋄ FIRST←Z←1
[21]  KRMS←RMS⋄ KPTP←PTP⋄ KMIN←MIN⋄ KMAX←MAX
    ∇


    ∇ FBKOUT;MUV;S;SEL
[1]   ⍝ USED FOR PLOTTING KINEMATIC PARAMETERS
[2]   ⍝ CALCULATED BY FBDVA.
[3]   GFCLG⋄ ⎕ARBOUT B1⋄ FBAXES
[6]   TRC←'AT'⋄ FBTXT
[8]   'PLEASE ENTER SELECTION:'
[9]   '   1) POS. OF JOINT 2'
[10]  '   2) VEL. OF JOINT 2'
[11]  '   3) ACC. OF JOINT 2'
[12]  '   4) POS. OF JOINT 3'
[13]  '   5) VEL. OF JOINT 3'
[14]  '   6) ACC. OF JOINT 3'
[15]  '   7) ANG. POS. OF LINK ',MUV←(1=FLAG23)⌽'23'
[16]  '   8) ANG. VEL. OF LINK ',MUV
[17]  '   9) ANG. ACC. OF LINK ',MUV
[18]  '  10) ANG. POS. OF LINK ',MUV←(1=FLAG43)⌽'43'
[19]  '  11) ANG. VEL. OF LINK ',MUV
[20]  '  12) ANG. ACC. OF LINK ',MUV
[21]  →(⎕LC[1]+4)×⍳((0=⍴⍴S)∧∧/(S←⎕)∊⍳12)
[22]  ⎕ARBOUT 7⋄ 'TRY AGAIN'⋄ →⎕LC[1]-3
[25]  SEL←('U2,[.1]V2',(4⍴' ')),[1]('U2D,[.1]V2D',(2⍴' ')),[.1
      ]'U2DD,[.1]V2DD'
[26]  SEL←SEL,[1]('U3,[.1]V3',(4⍴' ')),[1]('U3D,[.1]V3D',(2⍴'
      ')),[.1]'U3DD,[.1]V3DD'
[27]  SEL←SEL,[1]('PHI23',(8⍴' ')),[1]('PHI23D',(7⍴' ')),[.1]'
      PHI23DD',6⍴' '
[28]  SEL←SEL,[1]('PHI43',(8⍴' ')),[1]('PHI43D',(7⍴' ')),[.1]'
      PHI43DD',6⍴' '
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[29]  GFCLG◇ MUV←'[',((6<S)/'.'),'1]'
[31]  SEL←'PHI12,',MUV,,SEL[S;]
[32]  FNC←(-6<S)↓'PLOT3'◇ TRC←,AFLAG[S;]
[34]  ⍎'(10×S)',FNC,' ',SEL◇ AFLAG[S;]←'T '
    ∇

    ∇ FBMPOLIM;MUV
[1]    ⍝ CALC. LIMITS FOR LUMPED MASS PARAMS.
[2]    XO12MAX←(MUV←M12MIN,M12MAX)[1+X12MAX>0]×X12MAX÷A12
[3]    XO12MIN←MUV[1+X12MIN<0]×X12MIN÷A12
[4]    YO12MAX←MUV[1+Y12MAX>0]×Y12MAX÷A12
[5]    YO12MIN←MUV[1+Y12MIN<0]×Y12MIN÷A12
[6]    KO12MAX←M12MAX×((((|X12MIN)⌈|X12MAX)*2)+(((|Y12MIN)⌈|Y12
       MAX)*2)+K12MAX*2)÷A12*2
[7]    KO12MIN←M12MIN×((((|X12MIN)⌊|X12MAX)*2)+(((|Y12MIN)⌊|Y12
       MAX)*2)+K12MIN*2)÷A12*2
[8]    XO23MAX←(MUV←M23MIN,M23MAX)[1+X23MAX>0]×X23MAX÷A23
[9]    XO23MIN←MUV[1+X23MIN<0]×X23MIN÷A23
[10]   YO23MAX←MUV[1+Y23MAX>0]×Y23MAX÷A23
[11]   YO23MIN←MUV[1+Y23MIN<0]×Y23MIN÷A23
[12]   KO23MAX←M23MAX×((((|X23MIN)⌈|X23MAX)*2)+(((|Y23MIN)⌈|Y23
       MAX)*2)+K23MAX*2)÷A23*2
[13]   KO23MIN←M23MIN×((((|X23MIN)⌊|X23MAX)*2)+(((|Y23MIN)⌊|Y23
       MAX)*2)+K23MIN*2)÷A23*2
[14]   XO43MAX←(MUV←M43MIN,M43MAX)[1+X43MAX>0]×X43MAX÷A43
[15]   XO43MIN←MUV[1+X43MIN<0]×X43MIN÷A43
[16]   YO43MAX←MUV[1+Y43MAX>0]×Y43MAX÷A43
[17]   YO43MIN←MUV[1+Y43MIN<0]×Y43MIN÷A43
[18]   KO43MAX←M43MAX×((((|X43MIN)⌈|X43MAX)*2)+(((|Y43MIN)⌈|Y43
       MAX)*2)+K43MAX*2)÷A43*2
[19]   KO43MIN←M43MIN×((((|X43MIN)⌊|X43MAX)*2)+(((|Y43MIN)⌊|Y43
       MAX)*2)+K43MIN*2)÷A43*2
    ∇
    ∇ Z←FBOPT;PAIR;I;J;MUV;PQ
[1]    ⍝ PERFORMS DYNAMIC OPTIMIZATION
[2]    LIM←M23MIN,M23MAX,XO23MIN,XO23MAX
[3]    →(SYNCASE='2')/⎕LC[1]+5
[4]    YO12←SMCO[1]◇ YO23←SMCO[4]◇ YO43←SMCO[6]
[7]    →LB4
[8]    YO23←ITCO[2]◇ YO12←SFCO[3]+YO23
[10] LB4:GFCLA
[11]   'THE FREE MASS PARAMETERS FOR THE OPTIMIZATION'
[12]   'PROCESS THAT IS ABOUT TO BEGIN ARE:''M23'' AND ''XO23''
       '
[13]   'WHERE,'
[14]   (7⍴' '),(⍕LIM[1]),'≤M23≤',(⍕LIM[2]),' AND ',(⍕LIM[3]),'≤
       XO23≤',⍕LIM[4]
[15]   ⍳0
[16]   'PLEASE ENTER A PAIR OF NUMBERS INDICATING'
[17]   'HOW MANY EQUALLY SPACED VALUES IN THE ABOVE'
[18]   'INTERVALS YOU WANT TO CONSIDER, INITIALLY, FOR'
[19]   '''M23'' AND ''XO23'', IN THAT ORDER:'
[20]   →(⎕LC[1]+3)×⍳3=⍴1,PAIR←⎕
[21]   ⎕ARBOUT 7◇ →⎕LC[1]-2
[23]   ΔM23←(-/LIM[2 1])÷PAIR[1]-1
[24]   ΔXO23←(-/LIM[4 3])÷PAIR[2]-1
[25]   I←J←OF←0
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[26] LB1:MO23←LIM[1]+I×ΔM23
[27] LB2:XO23←LIM[3]+J×ΔXO23
[28]  →LB3×ι0=⍎'FINDM̲P̲',S̲Y̲N̲CASE
[29]  →LB3×ι0=⍎'CALCO̲F̲',S̲Y̲N̲CASE
[30]  →LB3×ιO̲F̲≥OF
[31]  O̲F̲←OF⋄ M̲23←MO23⋄ X̲O23←XO23⋄ K̲O23←KO23
[35]  X̲O12←XO12⋄ K̲O43←KO43⋄ X̲O43←XO43
[38] LB3:→LB2×ιPAIR[2]>J←J+1
[39]  →LB1×ιPAIR[1]>I←I+1+J←0
[40]  ⎕ARBOUT 7⋄ MUV←⎕DL 1⋄ ⎕ARBOUT 7
[43]  GFCLA⋄ →LB5×ιO̲F̲≠0
[45]  'SORRY, NO SOLUTION COULD BE FOUND.'
[46]  →LB6
[47] LB5:MO23←M̲23⋄ XO23←X̲O23⋄ KO23←K̲O23
[50]  XO12←X̲O12⋄ KO43←K̲O43⋄ XO43←X̲O43
[53]  'NO. OF DESIGN POINTS CONSIDERED=',⍕×/PAIR+1
[54]  'RANGES:',(⍕LIM[1]),'≤M23≤',(⍕LIM[2]),' AND ',(⍕LIM[3]),
      '≤XO23≤',⍕LIM[4]
[55]  'BEST SOLUTION:'
[56]  (15ρ' '),'SCORE=',⍕O̲F̲
[57]  (15ρ' '),'(IDEAL SOLUTION WOULD SCORE 100)'
[58]  (15ρ' '),'M23=',⍕MO23
[59]  (15ρ' '),'XO23=',⍕XO23⋄ ι0⋄ ι0
[62]  LIM←MO23+((-ΔM23),ΔM23)×MO23≠M23MIN,M23MAX
[63]  LIM←LIM,XO23+((-ΔXO23),ΔXO23)×XO23≠XO23MIN,XO23MAX
[64]  'PLEASE MAKE YOUR SELECTION:'
[65]  '  1) PERFORM ANOTHER SEARCH AROUND CURRENT OPTIMUM'
[66]  '  2) SHOW PLOT FOR DYNAMIC PROPERTIES'
[67]  →((0=ρρMUV)∧∧/MUV∊'12')↑(¯1+'12'ι1↑MUV←⍞)⌽LB4,LB7
[68]  ⎕ARBOUT 7⋄ →⎕LC[1]-2
[70] LB7:CASE←'T̲'
[71]  STADYNOUT
[72]  GFCLG⋄ DRWFB2⋄ 'THE OPTIMUM MASS PARAMETERS ARE:'⋄ ι0
[76]  (⍕ΔM12[1]),' ≤M12≤ ',⍕ΔM12[2]
[77]  'XO12=',⍕XO12⋄ 'YO12=',⍕YO12⋄ ι0⋄ MUV←∧/LINKS[2;1 2]≠LIN
      KS[2;3 4]
[81]  'M',(PQ←MUV⌽'23'),'=',⍕MO23
[82]  'X',PQ,'=',⍕(X̲23,423-X̲23)[MUV+1]
[83]  'Y',PQ,'=',⍕(Y̲23,-Y̲23)[MUV+1]
[84]  'K',PQ,'=',⍕K̲23⋄ ι0
[86]  (⍕ΔM43[1]),' ≤M43≤ ',(⍕ΔM43[2]),((ρMUV)×4=ρΔM43)↑MUV←'
      OR  ',(⍕(¯2↑ΔM43)[1]),' ≤M43≤ ',⍕(¯2↑ΔM43)[2]
[87]  'XO43=',⍕XO43⋄ 'YO43=',⍕YO43⋄ 'KO43=',⍕KO43⋄ ι0
[91]  'NOTE: K12 CAN ASSUME'
[92]  '       ANY ARBITRARY VALUE.'⋄ ι0
[94] LB6:'PLEASE, CHOOSE ONE ALTERNATIVE:'
[95]  '  1) RETROCEDE TO MODIFY MASS PARAMETERS'' LIMITS'
[96]  '  2) RETROCEDE TO REDO SYNTHESIS'
[97]  '  3) RETROCEDE TO MODIFY OBJECTIVE FUNCTION'
[98]  '  4) END'
[99]  →((0=ρρZ)∧∧/(Z←⍞)∊'1234')↑0
[100] ⎕ARBOUT 7⋄ →⎕LC[1]-2
    ∇
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    ∇ Z←FBSFCOEF;U;V;CO;IN
[1]    ⍝ CALC. COEFFS. FOR EQUALITY CONSTRAINTS
[2]    ⍝ IN S.FORCE SYNTHESIS.
[3]    IN←¯2↑IAIN
[4]    CO←(,SFL[;IN]-PRCPTSL)⌹(U,[1]V),(-V←V2DD[IN],[1.5]V3DD[I
       N]),[1]U←U2DD[IN],[1.5]U3DD[IN]
[5]     ⍝NOW NEC.COND. ARE GOING TO BE CHECKED
[6]    Z←0
[7]    →(((XO12MIN+M23MIN-XO23MAX)≤CO[1])∧CO[1]≤XO12MAX+M23MAX-
       XO23MIN)/0
[8]    →(((YO12MIN-YO23MAX)≤CO[3])∧CO[3]≤YO12MAX-YO23MIN)/0
[9]    →(((XO43MIN+XO23MIN)≤CO[2])∧CO[2]≤XO43MAX+XO23MAX)/0
[10]   →(((YO43MIN+YO23MIN)≤CO[4])∧CO[4]≤YO43MAX+YO23MAX)/0
[11]   KSFCO←SFCO◊ SFCO←CO
[13]   KCURVE←CURVE
[14]   SFI←-(2,N+1)⍴((U,[1]V),(-V←V2DD,[1.5]V3DD),[1]U←U2DD,[1.
       5]U3DD)+.×SFCO
[15]   CURVE←SFT←SFL+SFI
[16]   KPRCPTS←PRCPTSL
[17]   KIAIN←IAIN
[18]   KFIRST←FIRST
[19]   KR←R
[20]   FIRST←Z←1
    ∇


    ∇ Z←FBSMCOEF;U1;V1;U2;V2;U3;V3;U4;V4;IN;CO;D312;D412;D223;
      D323;D443;D343;D243
[1]    ⍝ CALC. COEFFS. FOR EQUALITY CONSTRAINTS
[2]    ⍝ IN S.MOMENT SYNTHESIS
[3]    PRCPTSL←PHI12[IN←1+⌊.5+PRCPTSL[1;]÷360÷N],[.5]PRCPTSL[2;
       ]
[4]    PRCPTSLD←(PRCPTSL[1;]×180÷○1),[.5]PRCPTSL[2;]
[5]    U1←U1-RPOINT[1]◊ V1←V1-RPOINT[2]
[7]    U2←U2-RPOINT[1]◊ V2←V2-RPOINT[2]
[9]    U3←U3-RPOINT[1]◊ V3←V3-RPOINT[2]
[11]   U4←U4-RPOINT[1]◊ V4←V4-RPOINT[2]
[13]   D312←(U1×U2DD)+V1×V2DD
[14]   D412←(U2×V2DD)-V2×U2DD
[15]   D223←((U2×V3DD)-V2×U3DD)+(U3×V2DD)-V3×U2DD
[16]   D323←((U2×U3DD)+V2×V3DD)-(U3×U2DD)+V3×V2DD
[17]   D443←(U3×V3DD)-V3×U3DD
[18]   D343←(U4×U3DD)+V4×V3DD
[19]   D243←(U4×V3DD)-V4×U3DD
[20]   CO←(SML[IN]-PRCPTSL[2;])⌹D312[IN],D412[IN],D223[IN],D323
       [IN],D443[IN],D343[IN],[1.5]D243[IN]
[21]    ⍝NOW NEC.CONDITIONS ARE GOING TO BE CHECKED
[22]   Z←0
[23]   →((YO12MIN≤CO[1])∧CO[1]≤YO12MAX)/0
[24]   →((((XO12MIN+M23MIN+KO23MIN)-2×XO23MAX)≤CO[2])∧CO[2]≤XO1
       2MAX+M23MAX+KO23MAX-2×XO23MIN)/0
[25]   →(((XO23MIN-KO23MAX)≤CO[3])∧CO[3]≤XO23MAX-KO23MIN)/0
[26]   →((YO23MIN≤CO[4])∧CO[4]≤YO23MAX)/0
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[27]  →(((KO23MIN+KO43MIN)≤CO[5])∧CO[5]≤KO23MAX+KO43MAX)/0
[28]  →((YO43MIN≤CO[6])∧CO[6]≤YO43MAX)/0
[29]  →(((XO43MIN-KO43MAX)≤CO[7])∧CO[7]≤XO43MAX-KO43MIN)/0
[30]  KSMCO←SMCO⋄ SMCO←CO
[32]  KCURVE←CURVE
[33]  SMI←(SMCO[1]×D312)+(SMCO[2]×D412)+(SMCO[3]×D223)+SMCO[4]
      ×D323
[34]  CURVE←SMT←SML+SMI←-SMI+(SMCO[5]×D443)+(SMCO[6]×D343)+SMC
      O[7]×D243
[35]  KPRCPTS←PRCPTSLD⋄ KFIRST←FIRST⋄ FIRST←Z←1
[38]  KRMS←RMS⋄ KPTP←PTP⋄ KMIN←MIN⋄ KMAX←MAX
    ∇


    ∇ Z←FINDMP1
[1]   ⍝ SOLVES EQUALITY CONSTRAINTS OBTAINED
[2]   ⍝ FROM S.MOMENT SYNTHESIS
[3]   Z←0
[4]   →((KO23MIN≤KO23)∧KO23MAX≥KO23←XO23-SMCO[3])/0
[5]   →((XO12MIN≤XO12)∧XO12MAX≥XO12←SMCO[2]+(2×XO23)-MO23+KO23
      )/0
[6]   →((KO43MIN≤KO43)∧KO43MAX≥KO43←SMCO[5]-KO23)/0
[7]   →((XO43MIN≤XO43)∧XO43MAX≥XO43←SMCO[7]+KO43)/0
[8]   Z←BASICMP
    ∇


    ∇ Z←FINDMP2
[1]   ⍝ SOLVES EQUALITY CONSTRAINTS OBTAINED
[2]   ⍝ FROM I.TOR+S.FOR SYNTHESIS.
[3]   Z←0
[4]   →((XO12MIN≤XO12)∧XO12MAX≥XO12←SFCO[1]+XO23-MO23)/0
[5]   →((XO43MIN≤XO43)∧XO43MAX≥XO43←SFCO[2]-XO23)/0
[6]   →((YO43MIN≤YO43)∧YO43MAX≥YO43←SFCO[4]-YO23)/0
[7]   →((KO23MIN≤KO23)∧KO23MAX≥KO23←XO23-ITCO[1])/0
[8]   →((KO43MIN≤KO43)∧KO43MAX≥KO43←ITCO[3]-KO23)/0
[9]   Z←BASICMP
    ∇


    ∇ Z←ITSMSYN;FLAG;MUV;MINPTS;KEEPSM;KCURVE;PRCPTS;KEEP;DIST
      ;IND;PRCPTSL;T;CASE;FIRST;DP;RMS;KRMS;PTP;KPTP;MAX;KMAX;
      MIN;KMIN
[1]   PERFORMS I.TOR OR S.MOMENT SYNTHESIS
[2]   TRC←'T'
[3]   CASE←14 ¯12[⍎SYNCASE]↑'SHAKING MOMENTINPUT TORQUE'
[4]   FIRST←Z←FLAG←0
[5]   MINPTS←(7 6 12 12[⍎M_TY],3 3 6 6[⍎M_TY])[⍎SYNCASE]
[6]   GFCLA
[7]   'TO PERFORM ',CASE,' SYNTHESIS'
[8]   'YOU MUST SELECT AT LEAST ',(⍕MINPTS),' PRECISION POINTS
      .'
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[9]   'IF MORE THAN ',(⍕MINPTS),' ARE SELECTED, THEY WILL'
[10]  'BE APPROXIMATED IN THE LEAST SQUARES SENSE.'
[11]  ⍳0⋄ 'PRESS ''RETURN'' TO CONTINUE.'⋄ MUV←⍞
[14]  KCURVE←⍳0
[15]  DP←2 ¯2[⍎SYNCASE]↑'SMIT'
[16]  ⍎DP,'CO←KCURVE'
[17]  CURVE←⍎(2 ¯2[⍎SYNCASE]↑'SMIT'),'T'
[18]  PRCPTSLD←PRCPTS←2 0⍴1
[19] LB2:GFCLG
[20]  444 PLOT PHI12,[.5]CURVE
[21]  PRCPTS←GFLTP PRCPTSLD⋄ GFACT 1⋄ DRWSTAR PRCPTS
[24]  5 60 GFTXT 'FOR THIS CURVE: RMS=',(⍕9 ¯3⍕RMS),' PTP=',⍕9
      ¯3⍕PTP
[25]  →(⎕LC[1]+3)×⍳0=⍴KCURVE
[26]  5 40 GFTXT 'FOR LAST CURVE: RMS=',(⍕9 ¯3⍕KRMS),' PTP=',⍕
      9 ¯3⍕KPTP
[27]  5 25 GFTXT(16⍴' '),'MIN=',(⍕9 ¯3⍕KMIN),' MAX=',⍕9 ¯3⍕KMA
      X
[28]  GFMOV 0 779⋄ ⎕ARBOUT 13
[30]  →LB3×⍳FLAG=0
[31]  FLAG←0⋄ 'DOES THIS CURVE SATISFY YOU?'
[33]  ⍎'→⎕LC+',1↑(¯1+'YN'⍳1↑⍞)⌽'341'
[34]  ⎕ARBOUT 7⋄ →⎕LC[1]-2⋄ →~Z←1
[37] LB3:'TO SELECT A POINT PRESS ANY KEY OTHER'
[38]  'THAN E,T,S,I OR Q, THEN PRESS RETURN.'⋄ ⍳0
[40]  'USE: ''E'' TO ELIMINATE A SELECTED POINT'
[41]  '      ''T'' TO THROW AWAY ALL CURRENTLY'
[42]  '           SELECTED POINTS'
[43]  '      ''S'' TO SYNTHESIZE ',CASE
[44]  '      ''I'' TO INTERCHANGE THIS CURVE WITH'
[45]  '           THE PREVIOUS ONE ',(19×0=⍴KCURVE)↑'(CAN''T BE
     USED NOW)'
[46]  '      ''Q'' TO QUIT SYNTHESIS AND RETROCEDE'
[47]  '           TO MODIFY INEQ.CONSTRAINTS.'⋄ ⍳0
[49]  'NOTE: 2 BEEPS MEANS NECESSARY CONDITIONS'
[50]  '       CANNOT BE SATISFIED.'
[51] LB4:⍎'→',3↑(¯3+3×102 106 117 114 116⍳¯1↑KEEP←GFGCUR)⌽'LB5
      LB9LB0000LB6LB7'
[52] LB5:→(⎕LC[1]+3)×⍳0≠¯1↑⍴PRCPTS
[53]  ⎕ARBOUT 7⋄ →LB4
[55]  IND←1↑⍋DIST←+⌿(PRCPTS-⍉(⌽⍴PRCPTS)⍴¯1↓KEEP)*2
[56]  →(⎕LC[1]+3)×⍳4≥DIST[IND]
[57]  ⎕ARBOUT 7⋄ →LB4
[59]  (,PRCPTS[;IND])GFCIRC 4 16
[60]  PRCPTS←0 ¯1↓IND⌽PRCPTS⋄ →LB4
[62] LB7:→(⎕LC[1]+3)×⍳(450≤KEEP[1])∧(KEEP[1]≤978)∧~KEEP[1]∊PRC
      PTS[1;]
[63]  ⎕ARBOUT 7⋄ →LB4
[65]  DRWSTAR 2↑KEEP
[66]  PRCPTS←PRCPTS,2↑KEEP⋄ →LB4
[68] LB6:→(⎕LC[1]+3)×⍳MINPTS≤¯1↑⍴PRCPTS
[69]  ⎕ARBOUT 7⋄ →LB4
[71]  GFACT 444⋄ PRCPTSL←GFPTL PRCPTS⋄ GFACT 1
[74]  →LB8×⍳1=⍎(4 2⍴'FBSCWAST')[⍎M_TY;],DP,'COEF'
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[75]  ⎕ARBOUT 7⋄ T←⎕DL 1⋄ ⎕ARBOUT 7⋄ →LB4
[79] LB8:FLAG←1⋄ →LB2
[81] LB0:PRCPTSLD←2 0⍴1⋄ →LB2
[83] LB9:→(⎕LC[1]+3)×⍳0≠⍴KCURVE
[84]  ⎕ARBOUT 7⋄ →LB4
[86]  MUV←CURVE⋄ CURVE←KCURVE⋄ KCURVE←MUV
[89]  MUV←FIRST⋄ FIRST←KFIRST⋄ KFIRST←MUV
[92]  MUV←RMS⋄ RMS←KRMS⋄ KRMS←MUV
[95]  MUV←PTP⋄ PTP←KPTP⋄ KPTP←MUV
[98]  MUV←MIN⋄ MIN←KMIN⋄ KMIN←MUV
[101] MUV←MAX⋄ MAX←KMAX⋄ KMAX←MUV
[104] MUV←⍎DP,'CO'⋄ ⍎DP,'CO','←K',DP,'CO'⋄ ⍎'K',DP,'CO←MUV'
[107] GFACT 444
[108] MUV←GFPTL PRCPTS⋄ PRCPTSLD←KPRCPTS⋄ KPRCPTS←MUV⋄ FLAG←FI
      RST⋄ →LB2
    ∇


    ∇ W←KEYBOARD INPANG;INPA;LI;LV;RI;RV;MUV
[1]    ⍝ USED FOR INPUT OF LOADS BY TYPING IN
[2]    ⍝ EACH INDIVIDUAL VALUE.
[3]    INPA←INPANG×180÷○1
[4]   LB5:GFCLA
[5]    W←⍳LV←0
[6]    LI←1
[7]   LB4:'YOU WANT TO CONSIDER THE INP ANGLE'
[8]    'FROM ',(⍕LV),' TO WHAT VALUE?(IN DEGREE)'
[9]    →(3+⎕LC[1])×⍳(0=⍴⍴RV)∧(∧/LV<RV)∧∧/(RV←⎕)≤360
[10]   ⎕ARBOUT 7
[11]   →⎕LC[1]-2
[12]   RV←INPA[RI←1↑⍋|INPA-RV]
[13]   'DOES THE PARAMETER TO BE INPUT'
[14]   'HAVE CONST. VALUE FOR THIS INTERVAL'
[15]   'OF THE INP. ANGLE?'
[16]   ⍎'→',5↑(¯5+5×'YN'⍳1↑⍞)⌽'LB1  LB2  ⎕LC+1'
[17]   ⎕ARBOUT 7
[18]   →⎕LC[1]-2
[19]  LB1:'WHAT IS THAT VALUE?'
[20]   W←W,(1+RI-LI)⍴1↑⎕
[21]   LI←RI+1
[22]   →LB3
[23]  LB2:'PLEASE ENTER VALUE OF PARAMETER'
[24]   'FOR INP ANGLE = ',⍕INPA[LI]
[25]   W←W,1↑⎕
[26]   →(⎕LC[1]-2)×⍳RI≥LI←LI+1
[27]  LB3:LV←INPA[LI-MUV←(⍴INPA)<LI]
[28]   →LB4×⍳MUV=0
[29]   →LB5×⍳∧/W=W[1]
[30]   TRC←'T'
[31]   GFCLG
[32]   7.89 PLOT INPANG,[.1]W
[33]   ⎕ARBOUT 13
[34]   'SELECT:'
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[35]  '  1) TO CONTINUE'
[36]  '  2) TO RE-ENTER DATA'
[37]  →((0=⍴⍴MUV)∧∧/MUV∊1 2)↑⍎5↑(¯5+5×1 2⍳MUV←⎕)⌽'⎕LC+3LB6  '
[38]  ⎕ARBOUT 7
[39]  →⎕LC[1]-2
[40]  GFDREG 7.89
[41] LB5:GFACT 1
[42]  GFCLA
[43]  GFMOV 0 779
[44]  ⎕ARBOUT 13
    ∇


    ∇ Z←LOADS;SH;I;MUV;LIN;II;GFLAG;WHILE
[1]   ⍝ HANDLES INPUT OF EXTERNAL LOADS
[2]   GFCLA
[3]   ⍎'DRW',(4 2⍴'FBSCWAST')[⍎M_TY;],'2'
[4]   SH←1↑⍴LINKS
[5]   'INDICATE ALL LINKS UNDER EXTERNAL LOADS:'
[6]   I←1
[7]   '  ',(⍕I),') LINK ',LINKS[I;3 4]
[8]   →(⎕LC[1]-1)×⍳SH≥I←I+1
[9]   '  ',(⍕1+SH),') THERE''S NO LINK UNDER LOAD'
[10]  →(⎕LC[1]+3)×⍳(∧/MUV=1+SH)∨∧/(MUV←⎕)∊⍳SH
[11]  ⎕ARBOUT 7⋄ →⎕LC[1]-2
[13]  LINKSUL←((⍳SH)∊MUV)/⍳SH
[14]  →((SH+1)∊MUV)↑Z←0
[15]  23⍴'*'⋄ '  1) FORCE ONLY'
[17]  '  2) MOMENT ONLY'
[18]  '  3) FORCE AND MOMENT'⋄ 23⍴'*'
[20]  'BASED ON OPTIONS ABOVE, INDICATE'
[21]  'KIND OF EXTERNAL LOAD'
[22]  LOKND←⍳0⋄ Z←I←1
[24] LB1:'FOR LINK ',(LINKS[LINKSUL[I];3 4]),':'
[25]  →(⎕LC[1]+3)×⍳(0=⍴⍴MUV)∧∧/(MUV←⎕)∊⍳3
[26]  ⎕ARBOUT 7⋄ →⎕LC[1]-2⋄ LOKND←LOKND,MUV
[29]  →LB1×⍳(⍴LINKSUL)≥I←I+1⋄ I←1
[31] LB7:LIN←LINKSUL[I]⋄ →LB2×⍳2=LOKND[I]
[33]  'FOR LINK ',(LINKS[LIN←LINKSUL[I];3 4]),', IS THE POINT
      OF APPLICATION'
[34]  'OF EXT FORCE CONSTANT?'
[35]  ⍎'→',5↑(¯5+5×'YN'⍳1↑⍞)⌽'LB3  LB4  ⎕LC+1'
[36]  ⎕ARBOUT 7⋄ →⎕LC[1]-2⋄LB3:⍳0
[39]  'ENTER THE X-Y COORDS OF THAT POINT:'
[40]  →(⎕LC[1]+3)×⍳3=⍴1,MUV←⎕⋄ ⎕ARBOUT 7⋄ →⎕LC[1]-2
[43]  ⍎'E',LINKS[LIN;1 2],'←2 1⍴',⍕TBL[LIN;2 3]+TBL[LIN;1]ROTV
      EC MUV
[44]  →LB5⋄LB4:II←1⋄ WHILE←(2,(N+1))⍴0⋄ ⍳0
[48]  'HOW DO YOU WANT TO ENTER THE'
[49]  ('XY'[II]),'-COORDINATES OF THE POINTS OF APPLICATION'
[50]  'OF EXT FORCE ON LINK',LINKS[LIN;3 4],'?'
[51]  '  1) CALCULATOR'
[52]  '  2) KEYBOARD'
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[53]  →(3+⎕LC[1])×⍳(0=⍴⍴MUV)∧∧/(MUV←⎕)∊1 2
[54]  ⎕ARBOUT 7⋄ →⎕LC[1]-2
[56]  WHILE[II;]←⍎(10↑(10×MUV-1)⌽'CALCULATORKEYBOARD  '),' PHI
      ',LINKS[1;1 2]
[57]  →(LB4+2)×⍳2=II←II+1
[58]  WHILE←(⍉((N+1),2)⍴TBL[LIN;2 3])+TBL[LIN;1]ROTVEC WHILE
[59]  ⍎'E',LINKS[LIN;1 2],'←WHILE'
[60] LB5:II←1⋄ WHILE←(2,(N+1))⍴0
[62]  'INDICATE COORD SYSTEM FOR THE'
[63]  'EXT FORCES (LINK',LINKS[LIN;3 4],') YOU''RE GOING TO EN
      TER:'
[64]  '  1) GLOBAL'
[65]  '  2) LOCAL'
[66]  →(⎕LC[1]+3)×⍳(0=⍴⍴GFLAG)∧∧/(GFLAG←⎕)∊1 2
[67]  ⎕ARBOUT 7⋄ →⎕LC[1]-2
[69] LB8:'HOW DO YOU WANT TO ENTER THE'
[70]  ((2 2⍴'UVXY')[GFLAG;II]),'-COMPONENT OF THE EXT FORCE (L
      INK',LINKS[LIN;3 4],')?'
[71]  '  1) CALCULATOR'
[72]  '  2) KEYBOARD'
[73]  →(3+⎕LC[1])×⍳(0=⍴⍴MUV)∧∧/(MUV←⎕)∊1 2
[74]  ⎕ARBOUT 7⋄ →⎕LC[1]-2
[76]  WHILE[II;]←⍎(10↑(10×MUV-1)⌽'CALCULATORKEYBOARD  '),' PHI
      ',LINKS[1;1 2]
[77]  →LB8×⍳2=II←II+1⋄ →(⎕LC[1]+2)×⍳(GFLAG=1)
[79]  WHILE←((180÷○1)×⍎'PHI',LINKS[LIN;1 2])ROTVEC2 TBL[LIN;1]
      ROTVEC WHILE
[80]  ⍎'F',LINKS[LIN;1 2],'←WHILE'
[81] LB2:→LB6×⍳1=LOKND[I]
[82]  'HOW DO YOU WANT TO ENTER THE'
[83]  'EXT MOMENT ON LINK',LINKS[LIN;3 4],'?'
[84]  '  1) CALCULATOR'
[85]  '  2) KEYBOARD'
[86]  →(3+⎕LC[1])×⍳(0=⍴⍴MUV)∧∧/(MUV←⎕)∊1 2
[87]  ⎕ARBOUT 7⋄ →⎕LC[1]-2
[89]  ⍎'T',LINKS[LIN;1 2],'←',⍕⍎(10↑(10×MUV-1)⌽'CALCULATORKEYB
      OARD  '),' PHI',LINKS[1;1 2]
[90] LB6:⍎(11×(⍴LINKSUL)≥I←I+1)↑'DRW',(4 2⍴'FBSCWAST')[⍎M_TY;]
      ,'2⋄→LB7'
   ∇


   ∇ MPIN;MUV;VAL
[1]   ⍝ HANDLES INPUT OF MASS PARAMETERS
[2]   GFCLG⋄ ⍎'DRW',(4 2⍴'FBSCWAST')[⍎M_TY;],'2'
[4]   I←1⋄ 'WHAT MASS PARAMETER'
[6]   'DO YOU WANT TO CHANGE?'
[7]   MUV←⍕(⍎'M',MUV),(⍎'X',MUV),(⍎'Y',MUV),⍎'K',MUV←LINKS[I;1
       2]
[8]   '  M',MUV,' X',MUV,' Y',MUV,' K',(MUV←LINKS[I;3 4]),'=',
      MUV
[9]   →(⎕LC[1]-2)×⍳(1↑⍴LINKS)≥I←I+1
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[10]  '(FOR EX: ENTER ''M',(LINKS[2;3 4]),''' TO ADRESS THE MA
      SS OF LINK ',LINKS[2;3 4]
[11]  'OR ANSWER ''NONE'' TO CONTINUE)'
[12] LB2:→(1=∧/'NONE'=4↑MUV←⍞)↑0
[13]  →(□LC[1]+2)×ι~∧/(¯2↑MUV)∊⍕ι9
[14]  →LB1×ι(4=ρ'1',MUV)∧((⍎¯2↑MUV)∊⍎,LINKS[;3 4],(1↑ρLINKS)ρ'
       ')∧(1↑MUV)∊'MXYK'
[15]  □ARBOUT 7◇ →□LC[1]-4
[17] LB1:'WHAT IS THE NEW VALUE OF ',MUV,'?'
[18]  →(□LC[1]+3)×ι0=ρρVAL←□
[19]  □ARBOUT 7◇ →□LC[1]-2
[21]  ⍎(1↑MUV),LINKS[(∧/LINKS[;3 4]=((1↑ρLINKS),2)ρ¯2↑MUV)ι1;1
       2],'←VAL'
[22]  'WHAT M.P. DO YOU WANT TO CHANGE NEXT?'◇ →LB2
    ∇


    ∇ MPLIMMOD;MUV;VAL
[1]    ⍝ ALLOWS MODIFICATION OF MASS PARAMETERS
[2]    GFCLG◇ ⍎'DRW',(4 2ρ'FBSCWAST')[⍎M_TY;],'2'
[4]    I←1
[5]    'WHAT LIMIT DO YOU WANT TO MODIFY?'◇ ι0
[7]    MUV←⍕(⍎'M',MUV),(⍎'X',MUV),(⍎'Y',MUV),⍎'K',MUV←LINKS[I;1
        2],'MIN'
[8]    '     M',MUV,' X',MUV,' Y',MUV,' K',(MUV←LINKS[I;3 4],'MI
       N'),'=',MUV
[9]    →(□LC[1]-2)×ι(1↑ρLINKS)≥I←I+1◇ ι0◇ I←1
[12]   MUV←⍕(⍎'M',MUV),(⍎'X',MUV),(⍎'Y',MUV),⍎'K',MUV←LINKS[I;1
        2],'MAX'
[13]   '     M',MUV,' X',MUV,' Y',MUV,' K',(MUV←LINKS[I;3 4],'MA
       X'),'=',MUV
[14]   →(□LC[1]-2)×ι(1↑ρLINKS)≥I←I+1
[15]   ι0◇ '(FOR EX: ENTER ''M',(LINKS[2;3 4]),'MIN''',' TO MOD
       IFY IT'
[17]   'OR ANSWER ''NONE'' TO CONTINUE)'
[18]  LB2:→(1=∧/'NONE'=4↑MUV←⍞)↑0
[19]   →(□LC[1]+2)×ι~(∧/(2↑1⌽MUV)∊'123456789')∧7=ρ'1',MUV
[20]   →LB1×ι((⍎2↑1⌽MUV)∊⍎,LINKS[;3 4],(1↑ρLINKS)ρ' ')∧((1↑MUV)
       ∊'MXYK')∧(∧/'MIN'=¯3↑MUV)∨∧/'MAX'=¯3↑MUV
[21]   □ARBOUT 7◇ →□LC[1]-4
[23]  LB1:'WHAT IS THE NEW VALUE OF ',MUV,'?'
[24]   →(□LC[1]+3)×ι0=ρρVAL←□
[25]   □ARBOUT 7◇ →□LC[1]-2
[27]   ⍎(1↑MUV),LINKS[(∧/LINKS[;3 4]=((1↑ρLINKS),2)ρ2↑1⌽MUV)ι1;
       1 2],(¯3↑MUV),'←VAL'
[28]   'WHAT LIMIT DO YOU WANT TO ALTER NEXT?'◇ →LB2
    ∇
```

```
    ∇ REG PLOT AO;KEEP;EQ;M;MUV
[1]   ⍝ PLOTS SCALAR FUNCTIONS OF INP.ANGLE
[2]   REG GFREG 450 195 978 585
[3]   REG GFSCL((×M)×(0,(-M),0,M←(8×0=MIN)+MIN)×EQ←MIN=MAX)+(0
      ,MIN←⌊/AO[2;]),360,MAX←⌈/AO[2;]
[4]   GFACT REG
[5]   KEEP←GVARG[10 11 12 13]
[6]   GVARG[10 11 12 13]←1 1 0 0
[7]   450 195 GFRECT 978 585
[8]   0 1 1 0 1 GFVABS⍉5 2⍴450 0 450 779 445 769 450 779 455 7
      69
[9]   0 1 1 0 1 GFVABS⍉5 2⍴450 195 1010 195 1000 200 1010 195
      1000 190
[10]  →L1×⍳(4>MUV)∨(MUV←⌊KEEP[4]+.5)>765
[11]  ⍎(23⍴(195≤MUV)∧(MUV≤585))/'(450,MUV)GFLINE 978,MUV'
[12]  (450,MUV)GFLINE 446,MUV
[13]  (450,(MUV-4),2)GFTXT '0'
[14] L1:450 581 11 GFTXT 9 ¯3⍕((EQ∧1=×MAX)∨~EQ)↑MAX+EQ×MAX××MA
      X
[15]  450 191 12 GFTXT 9 ¯3⍕((EQ∧¯1=×MIN)∨~EQ)↑MIN-EQ×MIN××MIN

[16]  1000 170 GFTXT 'IA'
[17]  ⎕ARBOUT B4
[18]  582 175 1 GFTXT '90'⋄ 719 175 2 GFTXT '180'
[20]  851 175 2 GFTXT '270'⋄ 983 175 2 GFTXT '360'
[22]  GVARG[10 11 12 13]←KEEP
[23]  (0,(¯1+¯1↑⍴AO)⍴1)GFVABS(AO[1;]×180÷○1),[.1]AO[2;]
[24]  PTP←MAX-MIN
[25]  RMS←((+/AO[2;]*2)÷¯1↑⍴AO)*.5
    ∇
```

```
    ∇ REG PLOT3 AUV;COL;MAG;KEEP;A;B;MIN
[1]   ⍝ PLOTS VECTORIAL FUNCTIONS OF INP.ANGLE
[2]   REG GFREG 800 220 980 400
[3]   REG GFSCL 0 0,R,R←⌈/MAG←(+⌿AUV[2 3;]*2)*.5
[4]   GFACT REG⋄ KEEP←GVARG[10 11 12 13]
[6]   GVARG[10 11 12 13]←1 1 0 0
[7]   0 1 1 0 1 GFVABS⍉5 2⍴615 220 1000 220 990 225 1000 220 9
      90 215
[8]   0 1 1 0 1 GFVABS⍉5 2⍴800 40 800 420 795 410 800 420 805
      410
[9]   795 410 1 GFTXT 'V'⋄ 990 200 GFTXT 'U'
[11]  800 220 GFCIRC 180⋄ GVARG[10 11 12 13]←KEEP
[13]  (0,(¯1+¯1↑⍴AUV)⍴1)GFVABS AUV[2 3;]
[14]  GVARG[10 11 12 13]←1 1 0 0
[15]  B←((-/AUV[2;A])ARG-/AUV[3;A←-\(¯1↑⍴AUV),1])+○(180+30 ¯30
      )÷180
[16]  1 0 1 GFVINC⌊.5+(A,(-A←(2○B[1]),1○B[1]),[1.1](2○B[2]),1○
      B[2])×10
[17]  →LB1×⍳R=MIN←⌊/MAG
[18]  800 20 12 GFTXT 'MIN. MAGNITUDE = ',8 ¯3⍕MIN
[19]  →2+⎕LC[1]
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[20] LB1:800 15 12 GFTXT 'MAGNITUDE=CONST.= ',8 ¯3⍕R
[21]  GVARG[10 11 12 13]←KEEP
[22]  (REG+1)GFREG 800 450 980 750
[23]  (REG+1)GFSCL 0 ¯360,R,0
[24]  GFACT REG+1◊ KEEP←GVARG[10 11 12 13]
[26]  GVARG[10 11 12 13]←1 1 0 0
[27]  620 450 GFRECT 980 750
[28]  0 1 1 0 1 GFVABS⍉5 2⍴800 750 800 430 805 440 800 430 795
       440
[29]  0 1 1 0 1 GFVABS⍉5 2⍴980 750 1000 750 990 745 1000 750 9
      90 755
[30]  990 730 GFTXT 'U'◊ 795 430 2 GFTXT 'IA'
[32]  (6⍴0 1)GFVABS⍉6 2⍴665 750 665 753 710 750 710 753 755 75
      0 755 753
[33]  (6⍴0 1)GFVABS⍉6 2⍴845 750 845 753 890 750 890 753 935 75
      0 935 753
[34]  980 760 4 GFTXT 8 ¯3⍕R◊ 797 760 GFTXT '0'
[36]  620 760 4 GFTXT 9 ¯3⍕-R◊ ⎕ARBOUT B2
[38]  610 745 1 GFTXT '0'◊ 610 670 2 GFTXT '90'
[40]  610 595 3 GFTXT '180'◊ 610 520 3 GFTXT '270'
[42]  610 445 3 GFTXT '360'◊ GVARG[10 11 12 13]←KEEP
[44]  (0,(¯1+COL←¯1↑⍴AUV)⍴1)GFVABS AUV[2;],[.1]-AUV[1;]←AUV[1;
      ]×180÷○1
[45]  (REG+2)GFREG 270 220 570 400
[46]  (REG+2)GFSCL 0 0 360,R◊ GFACT REG+2
[48]  KEEP←GVARG[10 11 12 13]
[49]  GVARG[10 11 12 13]←1 1 0 0
[50]  270 40 GFRECT 570 400
[51]  0 1 1 0 1 GFVABS⍉5 2⍴270 220 590 220 580 215 590 220 580
       225
[52]  0 1 1 0 1 GFVABS⍉5 2⍴270 400 270 420 275 410 270 420 265
       410
[53]  265 410 1 GFTXT 'V'◊ 580 200 GFTXT 'IA'
[55]  (6⍴0 1)GFVABS⍉6 2⍴270 265 267 265 270 310 267 310 270 35
      5 267 355
[56]  (6⍴0 1)GFVABS⍉6 2⍴270 175 267 175 270 130 267 130 270 85
       267 85
[57]  260 397 8 GFTXT 8 ¯3⍕R◊ 260 217 1 GFTXT '0'
[59]  260 37 9 GFTXT 9 ¯3⍕-R◊ ⎕ARBOUT B3
[61]  266 20 GFTXT '0'◊ 345 20 1 GFTXT '90'
[63]  425 20 2 GFTXT '180'◊ 500 20 2 GFTXT '270'
[65]  575 20 2 GFTXT '360'
[66]  GVARG[10 11 12 13]←KEEP
[67]  (0,(¯1+COL)⍴1)GFVABS AUV[1 3;]
    ∇
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    ∇ Z←ANG ROTVEC VEC;C;S;A
[1]   ⍝ ROTVEC - ROTATES VECTORS IN 'VEC' BY ANGLE IN 'ANG'.
[2]   ⍝ ANG    - POS.(CCW) OR NEG.(CW) ANGLE OF ROTATION IN D
     EGREE.
[3]   ⍝ VEC    - 2×N MATRIX OF N VECTORS. IF N=1, EITHER ⍴VEC
     =2 1
[4]   ⍝          OR ⍴VEC=2 IS OK.
[5]   ⍝ Z      - ROTATED VECTORS (⍴Z=⍴VEC).
[6]   Z←(2 2⍴C,(-S),(S←1○A),C←2○A←ANG×○÷180)+.×VEC
    ∇


    ∇ Z←A ROTVEC2 V;D;C;S;T;R;I
[1]   ⍝ ROTVEC2 - ROTATES EACH VECTOR IN 'V' BY CORRESPONDING

[2]   ⍝           ANGLE IN 'A'.
[3]   ⍝ A       - ROW VECTOR WITH ROTATION ANGLES IN DEGREE;
     ⍴A≥2 .
[4]   ⍝ V       - MATRIX WITH VECTORS TO BE ROTATED; ⍴V=2,⍴A
     .
[5]   ⍝ Z       - ROTATED VECTORS; ⍴Z=⍴V .
[6]   A←((D←⍴A),1 1)⍴A×○÷180
[7]   T←(C,-S),[2]((S←1○A),C←2○A)
[8]   Z←⍉+/T×V,[2]V←(D,1 2)⍴⍉V
    ∇


    ∇ Z←A ROTVEC3 V;D;C;S;T;R;I
[1]   ⍝ SAME  AS  ROTVEC2 EXCEPT ARGUMENT 'A' WHICH
[2]   ⍝ MUST BE IN RADIANS HERE.
[3]   A←((D←⍴A),1 1)⍴A
[4]   T←(C,-S),[2]((S←1○A),C←2○A)
[5]   Z←⍉+/T×V,[2]V←(D,1 2)⍴⍉V
    ∇


    ∇ WW←SCOREPLOT X;I;MUV;LINE
[1]   444 GFREG 600 200 1000 600
[2]   444 GFSCL 0 0 1 1
[3]   GFACT 1
[4]  LB1:GFCLG
[5]   600 620 GFLINE 600 200
[6]   600 200 GFLINE 1020 200
[7]   600 600 GFDALINE 1000 600
[8]   1000 600 GFDALINE 1000 200
[9]   1000 200 GFDALINE 600 600
[10]  1000 183 2 GFTXT 'WORST'
[11]  600 183 1 GFTXT '0'
[12]  800 183 3 GFTXT '←ACTUAL→'
[13]  600 595 2 GFTXT '1'
[14]  (600 455,MUV←1+¯1↑⍴X)GFTXT 'SCORE:'
[15]  (600 435,MUV)GFTXT X[1;]
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[16]  (600 415,MUV)GFTXT X[2;]
[17]  (600 395,MUV)GFTXT X[3;]
[18]  GFMOV 0 779⋄ ⎕ARBOUT 13⋄ I←LINE←0
[21] LB2:'INPUT A TRIAL VALUE FOR ''EXP'''
[22]  'TO SEE CORRESPONDING SCORE CURVE:'
[23]  →(⎕LC[1]+4)×⍳(2=⍴1,WW)∧0≤1↑WW←⎕
[24]  LINE←LINE+1
[25]  ⎕ARBOUT 7⋄ →⎕LC[1]-3
[27]  GFACT 444
[28]  (0,199⍴1)GFVABS ABSS,[.5](⌽ABSS←.005×⍳200)*WW
[29]  GFACT 1⋄ GFMOV 0 779
[31]  ⎕ARBOUT 13,(LINE←LINE+5)⍴10
[32]  'IS THIS VALUE OF ''EXP'' OK?'
[33]  →⍎5↑(¯5+5×'YN'⍳1↑MUV←⍞)⌽'00000⎕LC+4⎕LC+1'
[34]  LINE←LINE+1
[35]  ⎕ARBOUT 7⋄ →⎕LC[1]-3
[37]  GFMOV 0 779⋄ ⎕ARBOUT 13,(LINE←LINE+3)⍴10
[39]  →(LB2,LB1)[1+4=I←I+1]
    ∇


    ∇ SMRP;MUV
[1]   ⍳0
[2]   MUV←9 ¯12[⍎SYNCASE]↑'SYNTHESISOPTIMIZATION'
[3]   'PLEASE ENTER THE SHAKING MOMENT REFERENCE POINT'
[4]   'TO BE USED IN THE ',MUV,' PROCESS:'
[5]  LB1:→(⎕LC[1]+3)×⍳3=⍴1,RPOINT←⎕
[6]   ⎕ARBOUT 7⋄ →⎕LC[1]-2
[8]   →(('1'=SYNCASE)⍲(∧/RPOINT=U1,V1))↑LB2
[9]   'SORRY, CAN''T CHOOSE JOINT 1 TO BE'
[10]  'THE REFERENCE POINT.'⋄ →LB1
[12] LB2:FBSMI⋄ FBSML⋄ SMT←SML+SMI
    ∇


    ∇ SPEC
[1]   'CHOOSE UNIT FOR ANGULAR VELOCITY'
[2]   '   1) RADIANS PER SEC.'
[3]   '   2) RPM'
[4]   →(⎕LC[1]+3)×⍳(0=⍴⍴UNIT)∧∧/(UNIT←⍞)∊'12'
[5]   ⎕ARBOUT 7
[6]   →⎕LC[1]-2
[7]   'NOW ENTER INPUT ANGULAR VELOCITY'
[8]   '(+) IF CCW ; (-) IF CW:'
[9]   WI←⎕
[10]  'WHAT IS THE INPUT ANGLE INCREMENT IN DEGREE ?'
[11]  '(A SUBMULTIPLE OF 360 IS RECOMMENDED)'
[12]  N←⌊.5+360÷⎕
    ∇
```

```
    ∇ STADYNOUT;I;MUV;S;LL;FLAG;MUV2
[1]    TRC←'T'
[2]    MUV←1↑⍴JOINTS
[3]   LB1:GFCLG
[4]    ⍎'DRW',(4 2⍴'FBSCWAST')[⍎M_TY;],'2'
[5]    'WHAT VARIABLE DO YOU'
[6]    'WANT TO SEE PLOTTED?'
[7]    I←1
[8]    '  ',(⍕I),') FORCE AT JOINT ',(⍕I),' (ACTING UPON LINK '
       ,JOINTS[I;],')'
[9]    →(⎕LC[1]-1)×⍳MUV≥I←I+1
[10]   '  ',(⍕MUV+1),') SHAKING FORCE'
[11]   '  ',(⍕MUV+2),') INPUT TORQUE'
[12]   '  ',(⍕MUV+3),') SHAKING MOMENT'
[13]   '  ',(⍕MUV+4),') CONTINUE WITHOUT PLOTTING'
[14]   →(⎕LC[1]+3)×⍳(0=⍴⍴S)∧∧/(S←⎕)∊⍳MUV+4
[15]   ⎕ARBOUT 7
[16]   →⎕LC[1]-2
[17]   →(S=MUV+4)↑0
[18]   →(S≠MUV+3)↑LB2
[19]   'PLEASE ENTER THE U-V COORDS'
[20]   'OF REF. POINT FOR SHAKING MOMENT'
[21]   →(3=⍴1,RPOINT←⎕)↑⎕LC[1]+3
[22]   ⎕ARBOUT 7
[23]   →⎕LC[1]-2
[24]   ⍎(4 2⍴'FBSCWAST')[⍎M_TY;],'SML'
[25]   →LB5×⍳'L'∊CASE
[26]   ⍎(4 2⍴'FBSCWAST')[⍎M_TY;],'SM',MUV2←'II'[1+'T'∊CASE]
[27]   ⍎'SM',CASE,'←',⍕SML+⍎'SM',MUV2
[28]  LB5:GFCLG
[29]   444 PLOT(⍎'PHI',LINKS[1;1 2]),[.5]⍎'SM',CASE
[30]   GFACT 1
[31]   500 620 GFTXT 'RMS=',(⍕9 ¯3⍕RMS),' PTP=',⍕9 ¯3⍕PTP
[32]   →LB4
[33]  LB2:→(S≠MUV+2)↑LB3
[34]   GFCLG
[35]   444 PLOT(⍎'PHI',LINKS[1;1 2]),[.5]⍎'IT',CASE
[36]   GFACT 1
[37]   500 620 GFTXT 'RMS=',(⍕9 ¯3⍕RMS),' PTP=',⍕9 ¯3⍕PTP
[38]   →LB4
[39]  LB3:→(LB4-2)×⍳~(∧/∧⌿(⍎'SF',CASE)=0)∧S=MUV+1
[40]   GFCLA
[41]   'SHAKING FORCE = ZERO'
[42]   →LB4
[43]   GFCLG
[44]   444 PLOT3(⍎'PHI',LINKS[1;1 2]),[1]⍎3↑(3×S=MUV+1)⌽'F',(⍕S
       ),CASE,'SF',CASE
[45]  LB4:GFACT 1
[46]   GFMOV 0 779
[47]   ⎕ARBOUT 13 10 10
[48]   'PRESS ''RETURN'' TO CONTINUE'
[49]   LL←⍞◇ →LB1
    ∇
```

```
      ∇ WFWAID
       →(□LC[1]+3)×ι(0<¯1↑MUV)∧(3=ρ1,MUV)∧(10≥1↑MUV)∧0≤1↑MUV←□
       □ARBOUT 7
       →□LC[1]-2
       WF←WF,MUV[1]
       WORST←WORST,MUV[2]
      ∇
```

```
      ∇ WFWIN;NJ;I;MUV;Z;WF
       GFCLA
       WF←WORST←ι0
       NJ←1↑ρJOINTS
       I←1
       'INPUT WEIGHTING FACTOR (0≤WF≤10) AND WORST VALUE (IN TH
       AT ORDER) FOR:'
      LB1:'MAX MAGNITUDE OF FORCE AT JOINT ',⍕I
       WFWAID
       →LB1×ιNJ≥I←I+1
       →LB2×ιSYNCASE='2'
       'MAX MAGNITUDE OF SHAKING FORCE'
       WFWAID
       'PEAK-TO-PEAK VALUE OF INPUT TORQUE'
       WFWAID
       'RMS VALUE OF INPUT TORQUE'
       WFWAID
       'MAX ABS VALUE OF INPUT TORQUE'
       WFWAID
       →LB3
      LB2:'PEAK-TO-PEAK VALUE OF SHAKING MOMENT'
       WFWAID
       'RMS VALUE OF SHAKING MOMENT'
       WFWAID
       'MAX ABS VALUE OF SHAKING MOMENT'
       WFWAID
      LB3:WF←(Z←WF≠0)/WF
       WF←Z\100×WF÷+/WF
      ∇
```

```
      ∇ MPLIMITS;I;J;PAIR;VAR
       GFCLG
       ⍎'DRW',(4 2ρ'FBSCWAST')[⍎M_TY;],'2'
       J←I←1
       'PLEASE ENTER AN ORDERED PAIR OF NUMBERS'
       'FOR ''MIN'' AND ''MAX'' VALUES OF:'
      LB1:(VAR←'MXYK'[J]),LINKS[I;3 4]
       →(□LC[1]+3)×ι3=ρ1,PAIR←□
       □ARBOUT 7
       →□LC[1]-2
       ⍎VAR,LINKS[I;1 2],'MIN←',⍕PAIR[1]
       ⍎VAR,LINKS[I;1 2],'MAX←',⍕PAIR[2]
       →LB1×ι4≥J←J+1
       →LB1×ι(1↑ρLINKS)≥I←I+J←1
      ∇
```

```
     ∇ Z←SFSYN;MUV;FLAG;FIRST;MINPTS;KCURVE;PRCPTS;PRCPTSL;LINE
        ;IAIN;KIAIN;KFIRST;R;KR
[1]    GFCLA◇ TRC←'T'◇ FIRST←Z←FLAG←0
[4]    'DO YOU WANT COMPLETE SHAKING FORCE BALANCE?'
[5]    →((□LC[1]+3),LB1,□LC[1]+1)['YN'⍳1↑⍞]
[6]    □ARBOUT 7◇ →□LC[1]-2
[8]    →(((XO12MIN+M23MIN-XO23MAX)>0)∨0>XO12MAX+M23MAX-XO23MIN)
       /LB
[9]    →(((YO12MIN-YO23MAX)>0)∨0>YO12MAX-YO23MIN)/LB
[10]   →(((XO43MIN+XO23MIN)>0)∨0>XO43MAX+XO23MAX)/LB
[11]   →(((YO43MIN+YO23MIN)>0)∨0>YO43MAX+YO23MAX)/LB
[12]   SFCO←0 0 0 0◇ →(Z←1)/0
[14]  LB:'SORRY THE INEQ. CONSTRAINTS ON THE'
[15]   'MASS PARAMETERS CANNOT BE SATISFIED.'
[16]   'PLEASE, SELECT AN ALTERNATIVE:'
[17]   '  1) RETROCEDE TO MODIFY INEQ. CONSTRAINTS'
[18]   '  2) CHANGE TO PARTIAL SYNTHESIS'
[19]   →((0=⍴⍴MUV)∧∧/MUV∊'12')↑(¯1+'12'⍳1↑MUV←⍞)⌽0,LB1,□LC[1]+1

[20]   □ARBOUT 7◇ →□LC[1]-2
[22]  LB1:MINPTS←2 2 3 3[⍎M_TY]
[23]   GFCLA
[24]   '  TO PERFORM SHAKING FORCE SYNTHESIS USE'
[25]   '  CROSS HAIRS TO FIND INPUT ANGLE (IN DEGREE)'
[26]   '  CORRESPONDING TO THE HODOGRAPH''S POINT'
[27]   '  YOU WANT TO SHIFT. PRESS ''A'', TYPE IN'
[28]   '  THE VALUE FOR THAT ANGLE, THEN USE'
[29]   '  CROSS HAIRS INTERSECT TO INDICATE THE'
[30]   '  NEW DESIRABLE LOCATION OF POINT.'
[31]   '  REPEAT THIS PROCESS FOR A SECOND POINT.'◇ ⍳0
[33]   'NOTE: TWO BEEPS MEANS NECESSARY CONDITIONS'
[34]   '        CANNOT BE SATISFIED.'◇ ⍳0◇ MUV←⍞
[37]   KCURVE←SFCO←2 0⍴1
[38]   CURVE←SFT◇ IAIN←¯1
[40]   PRCPTSL←PRCPTS←2 0⍴1
[41]  LB2:GFCLG
[42]   444 PLOT3 PHI12,[1]CURVE
[43]   GFACT 444
[44]   PRCPTS←GFLTP PRCPTSL
[45]   GFACT 1◇ DRWSTAR PRCPTS
[47]   GFMOV 0 779◇ □ARBOUT 13◇ LINE←0
[50]   →(□LC[1]+4)×⍳∧/2 0=⍴KCURVE
[51]   'FOR LAST CURVE: MAX. MAG.=',(⍕8 ¯3⍕KR)◇ ⍳0◇ LINE←2
[54]   →LB3×⍳FLAG=0◇ FLAG←0
[56]   'DOES THIS CURVE SATISFY YOU?'◇ LINE←LINE+1
[58]   ⍎'→□LC[1]+',1↑(¯1+'YN'⍳1↑⍞)⌽'451'
[59]   □ARBOUT 7◇ LINE←LINE+1◇ →□LC[1]-3◇ →~Z←1
[63]  LB3:'USE: ''A''-TO TYPE IN ANGLE'
[64]   '       ''T''-TO THROW AWAY POINTS'
[65]   '       ''S''-TO SYNTHESIZE'
[66]   '       ''I''-TO INTERCHANGE'
[67]   '       ''Q''-TO QUIT SYNTHESIS'
[68]   '           AND RETROCEDE'
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[69]  '  OTHERS-TO SELECT NEW LOCATION'
[70]  '          FOR A HODOGRAPH''S POINT'⋄ LINE←LINE+8
[72] LB4:⍎'→',3↑(¯3+3×98 106 117 114 116⍳¯1↑KEEP←GFGCUR)⌽'LB5L
      B9LB0000LB6LB7'
[73] LB5:⍎'→⎕LC[1]+','31'[1+((ρ1,IAIN)≠2+MUV)∨2=MUV←¯1↑ρPRCPTS
      ]⋄ ⎕ARBOUT 7⋄ →LB4
[76]  GFMOV 0 779⋄ ⎕ARBOUT 13,LINEρ10
[78]  →((KEEP≠¯1↑IAIN)∧(1≤KEEP)∧(N+1)≥KEEP←1+⌊.5+(1↑⎕)÷360÷N)/
      ⎕LC[1]+4
[79]  ⎕ARBOUT 7⋄ LINE←LINE+2⋄ →⎕LC[1]-3
[82]  IAIN←IAIN,KEEP⋄ LINE←LINE+2⋄ →LB4
[85] LB9:→((0≠¯1↑ρKCURVE)∧(ρ1,IAIN)=2+MUV←¯1↑ρPRCPTS)/⎕LC[1]+3

[86]  ⎕ARBOUT 7⋄ →LB4
[88]  MUV←CURVE⋄ CURVE←KCURVE⋄ KCURVE←MUV
[91]  MUV←FIRST⋄ FIRST←KFIRST⋄ KFIRST←MUV
[94]  MUV←R⋄ R←KR⋄ KR←MUV
[97]  MUV←SFCO⋄ SFCO←KSFCO⋄ KSFCO←MUV
[100] GFACT 444
[101] MUV←GFPTL PRCPTS⋄ PRCPTSL←KPRCPTS⋄ KPRCPTS←MUV
[104] MUV←IAIN⋄ IAIN←KIAIN⋄ KIAIN←MUV⋄ →LB2
[108]LB0:PRCPTSL←2 0ρ1⋄ IAIN←¯1⋄ →LB2
[111]LB7:→(((MUV+3)=ρ1,IAIN)∧∨/0 1=MUV←¯1↑ρPRCPTS)/⎕LC[1]+3
[112] ⎕ARBOUT 7⋄ →LB4
[114] DRWSTAR 2↑KEEP⋄ PRCPTS←PRCPTS,2↑KEEP⋄ →LB4
[117]LB6:→(MINPTS=¯1↑ρPRCPTS)/⎕LC[1]+3
[118] ⎕ARBOUT 7⋄ →LB4
[120] GFACT 444⋄ PRCPTSL←GFPTL PRCPTS⋄ GFACT 1
[123] →LB8×⍳1=⍎(4 2ρ'FBSCWAST')[⍎M_TY;],'SFCOEF'
[124] ⎕ARBOUT 7⋄ T←⎕DL 1⋄ ⎕ARBOUT 7⋄ →LB4
[128]LB8:FLAG←1
[129] →LB2
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I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

Delbert Tesar, Chairman
Graduate Research Professor of Mechanical Engineering

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George N. Sandor
Research Professor of Mechanical Engineering

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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Professor of Mechanical Engineering

I certify that I have read this study and that in my opinion it conforms to acceptable standards of scholarly presentation and is fully adequate, in scope and quality, as a dissertation for the degree of Doctor of Philosophy.

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